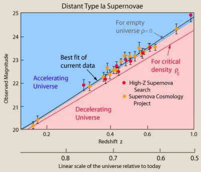
Solar system tests and chameleon effect in $f(R)$ gravity

Susana J. Landau[1] Lucila Kraiselburd [2] Carolina Negrelli[2]
Marcelo Salgado[3]

[1]Instituto de Física de Buenos Aires - CONICET - Universidad de Buenos Aires
[2]Facultad de Ciencias Astronómicas y Geofísicas - Universidad de La Plata
[3]Instituto de Ciencias Nucleares - Universidad Nacional Autónoma de México

Dark Side of the Universe - Buenos Aires - July 2019



- In 1998 two independent research teams (Supernova Cosmology Project and High-Z SN search) reported strong evidence that the late universe expansion is accelerating.
- Data from the Cosmic Microwave Background and Large Scale Structure confirm the late-time acceleration of the universe.
- This resulted in a modification of the standard cosmological model: The Λ_{CDM} model where a constant is added to Einstein's equation.

Alternative models to explain the late-time acceleration of the universe:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 4\pi G T_{\mu\nu}$$

- Scalar Fields minimally coupled to matter and gravity
  - quintessence
  - k-essence
- Scalar Fields with non-minimal coupling to matter:
  - dilatons, symmetrons: coupling to matter depends on the environment
  - chameleons: *thin shell* effect; coupling to matter depends on the mass $m(\phi)$.
  - galileons, beyond Horndesky: Vainstein Mechanism
- Alternative theories of gravity: $f(R)$, massive gravity

Outline

- Observational constraints on the posnewtonian parameter γ
- Basics of f(R) theories
- Our approach to the Solar System Problem
- Results
- Conclusions

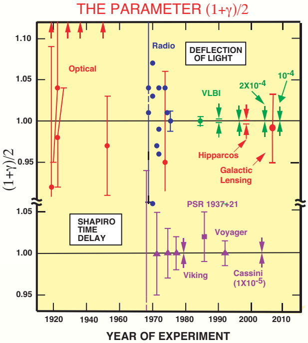
Observational constraints on the PPN parameter in the Solar System

A static spherically symmetric metric can be written as:

$$ds^2 = -[1 - 2\Phi(r)]dt^2 + [1 + 2\Psi(r)]dr^2 + r^2 d\Omega^2 \,.$$

and the corresponding posnewtonian parameter (PPN) γ can be expressed as:

$$\gamma(r) = \frac{\Psi(r)}{\Phi(r)}$$



Solar Sistem Tests: Time delay of light

A radar signal sent across the solar system past the Sun to a planet or satellite and returned to the Earth suffers an additional non-Newtonian delay in its round-trip travel time:

$$\delta t = 2(1+\gamma)M_\odot \ln\left(\frac{(r_\oplus + \mathbf{x}_\oplus \cdot \mathbf{n})(r_\mathrm{e} - \mathbf{x}_\mathrm{e} \cdot \mathbf{n})}{d^2}\right),$$

where $\mathbf{x}_\mathrm{e}$ $(\mathbf{x}_\oplus)$ are the vectors, and r_e $(r_\oplus)$ are the distances from the Sun to the source (Earth), respectively.

Basics of $f(R)$ theories

The action in $f(R)$ gravity can be expressed:

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\,f(R) + \int \mathrm{d}^4x\mathcal{L}_M(g_{\mu\nu},\Psi_M)\,,$$

where $\kappa^2 = 8\pi G$, g is the determinant of the metric $g_{\mu\nu}$, and $\mathcal{L}_M$ is a matter Lagrangian. The Ricci scalar R is defined by $R = g^{\mu\nu}R_{\mu\nu}$.
The field equation can be derived by varying the action with respect to $g_{\mu\nu}$:

$$f_R(R)R_{\mu\nu}(g) - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu f_R(R) + g_{\mu\nu}\Box f_R(R) = \kappa^2 T^{(M)}_{\mu\nu}$$

where $f_R(R) \equiv \partial f/\partial R$. $T^{(M)}_{\mu\nu}$ is the energy-momentum tensor of the matter fields defined by the variational derivative of $\mathcal{L}_M$ in terms of $g^{\mu\nu}$:

$$T^{(M)}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\mathcal{L}_M}{\delta g^{\mu\nu}}\,.$$

We assume a static spherical symetrical space-time:

$$ds^2 = -[1-2\Phi(r)]dt^2 + [1+2\Psi(r)]dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

The equations for the metric perturbation $\Phi(r)$ and $\Psi(r)$:

$$\begin{aligned}
\Psi' &= \frac{1+2\Psi}{r(2f_R + rR'f_{RR})}\Bigg\{4f_R\Psi - 2(1+2\Psi)r^2\kappa T^t_t \\
&+\frac{(1+2\Psi)r^2}{3}(Rf_R + f + 2\kappa T) + \frac{rR'f_{RR}}{f_R}\Big[\frac{(1+2\Psi)r^2}{3}(2Rf_R - f + \kappa T) \\
&-\kappa(1+2\Psi)r^2(T^t_t + T^r_r) + 4\Psi f_R + 2rR'f_{RR}\Big]\Bigg\} \\
\Phi' &= \frac{1-2\Phi}{r(2f_R + rR'f_{RR})}\Big[(1+2\Psi)r^2(f - Rf_R + 2\kappa T^r_r) \\
&+4f_R\Psi - 4rR'f_{RR}\Big],
\end{aligned}$$

where $f_{RR} = \partial^2 f/\partial R^2$.

Equation for $R(r)$

In a Minkowsky background:

$$R'' + \frac{2R'}{r} = \frac{1}{3f_{RR}}\Big(\kappa T + 2f - Rf_R - 3f_{RRR}R'^2\Big) \ ,$$

Previous analyses rely in :

- Defining a new scalar field $\chi = f_R$:

$$\chi'' + \frac{2\chi'}{r} = \frac{1}{3}\Big(\kappa T + 2f - R\chi\Big) \ = \frac{dV_{\text{eff}}}{dR}$$

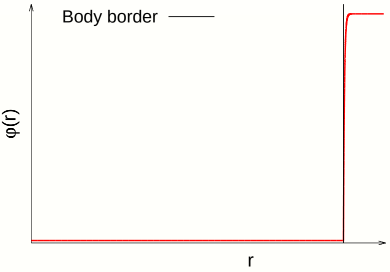
 where R and $f(R)$ are implicit functions of χ.

 .

- Defining a new chameleon like scalar field φ, where $\chi = \exp[-\varphi/(\sqrt{6}M_p)]$ and a conformal Einstein frame metric.

The solution for the chameleon field

- The solution of the chameleon field for one body is a step function, where the value of the field is equal to the minimum of the potential in each medium.



Our approach

We consider the following model for the Sun:

$$\rho(r) = \begin{cases} \rho_\odot = 1.43 \text{ g cm}^{-3} & (0 \leq r \leq \mathcal{R}_\odot) \\ \\ \rho_{\rm cor} = 10^{-15} \text{ g cm}^{-3} & (\mathcal{R}_\odot \leq r \leq \mathcal{R}_{\rm cor}) \\ \\ \rho_{\rm IM} = 10^{-24} \text{ g cm}^{-3} & (\mathcal{R}_{\rm cor} \leq r \leq \mathcal{R}_{IM}) \end{cases}$$

with $\mathcal{R}_{IM} \sim 150\,\text{A.U.}$ and $\mathcal{R}_{\rm cor} = 15\mathcal{R}_\odot$.

Our approach for solving the equation for $R(r)$

Neglecting the term with R'^2:

$$R'' + \frac{2R'}{r} = \frac{dV_{\rm eff}}{dR} = \frac{\kappa T + 2f - Rf_R}{3f_{RR}}$$

We will aproximate the r.h.s of the last equation, around the mininum of the effective potential in each region as follows:

$$R_I'' + \frac{2R_I'}{r} \approx m^2_{{\rm eff},I}(R_I - R^I_{\rm min}) \,,$$

where the index I stands for "in", "cor" and "out", and

$$m^2_{\rm eff} \quad = \tfrac{\kappa T + 2f - R^2 f_{RR}}{3Rf_{RR}}|_{R_{\rm min}}.$$

Our approach for solving the equation for $R(r)$

Accordingly, we propose the following solution:

$$R(r) = \begin{cases} R_{\rm in}(r) = R^{\rm in}_{\rm min} + \tilde{R}_{\rm in}(r) & (0 \leq r \leq \mathcal{R}_\odot) \\ \\ R_{\rm cor}(r) = R^{\rm cor}_{\rm min} + \tilde{R}_{\rm cor}(r) & (\mathcal{R}_\odot \leq r \leq \mathcal{R}_{\rm cor}) \\ \\ R_{\rm IM}(r) = R^{\rm IM}_{\rm min} + \tilde{R}_{\rm IM}(r) & (\mathcal{R}_{\rm cor} \leq r \leq \mathcal{R}_{IM}) \end{cases}$$

where we assume $\tilde{R}_{\rm in}(r) \ll R^{\rm in}_{\rm min}$, $\tilde{R}_{\rm IM}(r) \ll R^{\rm IM}_{\rm min}$, and $\tilde{R}_{\rm cor}(r) \ll R^{\rm cor}_{\rm min}$.
To obtain the posnewtonian parameter γ we must:

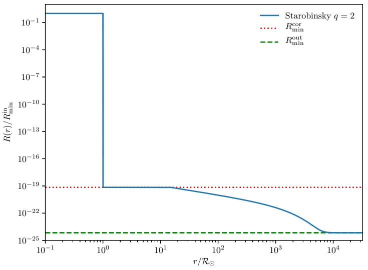
- Insert the analytical solution of $R(r)$ in the equations of the metric perturbations $\Phi(r)$ and $\Psi(r)$.
- Linearize the equations for the metric perturbations and solve them.

$R(r)$ for the Starobinsky model

Starobinsky JETP Lett. 86,157 (2007)

$$f(R) = R + \lambda R_s \left[\left(1 + \frac{R^2}{R_s^2} \right)^{-q} - 1 \right]$$

We fix $R_s = 4.17 H_0^2$ and $\lambda = 1$.



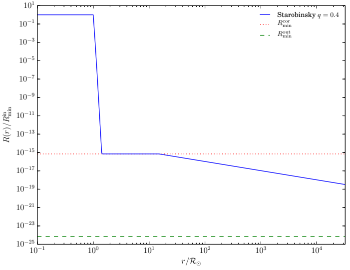


FIGURE: Left: $q = 2$ Right: $q = 0.4$. The red dotted line and the green dashed line indicate the values $R_{\rm min}^{\rm cor}$ and $R_{\rm min}^{\rm IM}$, respectively.

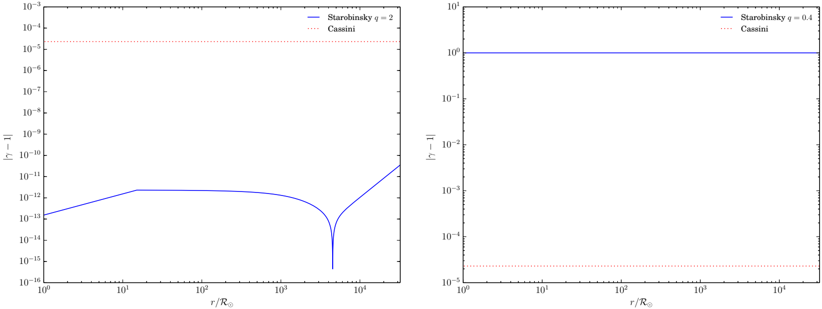


FIGURE: Deviation parameter $|\gamma - 1|$ as a function of r (in units of $\mathcal{R}_\odot$) computed from the Starobinsky model. Left: $q = 2$ Right: $q = 0.4$. The constraint from the Cassini mission is shown as a horizontal dotted red line.

$R(r)$ and $\gamma(r)$ for the Miranda model

Miranda et al. PRL 102, 221101 (2009)

$$f(R) = R - \alpha R_{\rm m} \ln\left(1 + \frac{R}{R_{\rm m}}\right),$$

with $R_{\rm m} = H_0^2$ and $\alpha = 2.0$.

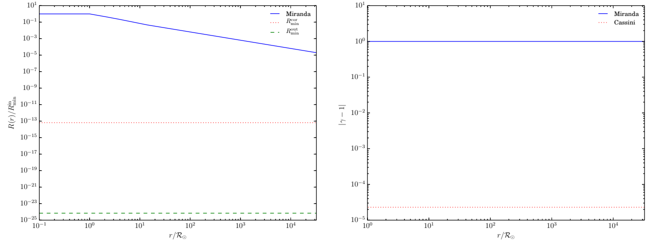


FIGURE: Left: R(r) for the Miranda Model Right: Deviation parameter $|\gamma - 1|$ as a function of r (in units of $\mathcal{R}_\odot$). The constraint from the Cassini mission is shown as a horizontal dotted red line.

Results for other models

We also tested:

- The Hu-Sawicky model (Hu & Sawicky PRD 76, 66400 (2007)):

$$f(R) = R - m^2 \frac{c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1} ,$$

  We assume $m^2 = 0.24 H_0^2$, $c_1 = 1.25 \times 10^{-3}$, $c_2 = 6.56 \times 10^{-5}$.

- The exponential model (Kerner et al PRD 77, 046009 (2009)):

$$f(R) = R - \beta R_* (1 - e^{-R/R_*}) ,$$

We assume $R_* = 2.5 H_0^2$ and $\beta = 2.0$.

- The behaviour of the exponential model and the Hu-Sawicky model for $n \geq 4$ is the same as that for the Starobinsky model for $q \geq 2$.
- The behaviour of the Hu-Sawicky model for $n < 4$ is the same as that for the Starobinsky model for $q < 2$.

Summary and conclusions

- We proposed a new approach to obtain an expresssion for the PPN γ for $f(R)$ theories in the Solar System.
- We applied this approach to several $f(R)$ models and work in the Jordan frame.
- The Starobinksy model for $q \geq 2$, the Hu-Sawicky model for $n \geq 4$ and the exponential model are not ruled out by the Solar Systems estimates of the PPN parameter γ.
- The Starobinksy model for $q < 2$, the Hu-Sawicky model for $n < 4$ and the Miranda model predictions are not consistent with the bounds obtained by the Casini mission and therefore must be ruled out.
- Our results are coincident with previous results in the literature.

Thank you !!!! Muchas gracias!!!