Constrained analytic model of Galactic dark matter subhalos

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Based on works with Gaétan Facchinetti, Thomas Lacroix, Martin Stref, et al. (1610.02233, 1805.02402, 1904.10935, 1905.02008 + work in prep)

Dark Side of the Universe

UBA, Buenos Aires – July 18, 2019
* Motivations

* Roadmap for a consistent model + some results

* Perspectives
Core/cusp problem ↔ regularity vs. diversity problems.
Maybe baryonic effects. Important to clarify.

CDM issues on small (subgalactic) scales

Small-Scale Challenges to the ΛCDM Paradigm

James S. Bullock\textsuperscript{1} and Michael Boylan-Kolchin\textsuperscript{2}

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See also NFW’s talks

Tulin+18 after Oman+15
Diversity problem
**CDM issues on small (subgalactic) scales**

Small-SCALE CHALLENGES TO THE ΛCDM PARADIGM

James S. Bullock¹ and Michael Boylan-Kolchin²

Has motivated pure DM solutions: eg ULA, SIDM

→ probes on small scales important tests for all DM scenarios

NB: baryonic physics does matter anyway!

Core/cusp problem ↔ regularity vs. diversity problems.

Maybe baryonic effects. Important to clarify.

See also NFW’s talks
A test of dark matter-only structuring properties: Dark subhalos

Proving/excluding the existence of dark matter subhalos?
* deep implication for dark matter scenarios + cosmology
* access to both DM candidate properties and primordial power spectrum
* independent test of “dark matter solutions” to the current small-scale issues

Looking for CDM subhalos in the Milky Way?
=> need for an accurate and dynamically consistent population model (MW=strongly constrained system)
Looking for / impact of dark matter subhalos

1. Particle dark matter searches

**Direct searches (WIMPs or axions) + solar neutrinos:**
→ (large) fluctuations in local density (A. Ibarra’s talk)
→ streams in local velocity distribution (S. White’s talk)

**Indirect searches:**
→ boost in the annihilation rate (S. White’s talk)
→ impact on v-dependent signatures
→ individual sources e.g. in gamma-rays

**Interaction with stars:**
→ DM capture enhanced (A. Ibarra’s talk)
Looking for / impact of dark matter subhalos
2. Gravitational searches

++ astrometry + lensing (micro/weak/strong) + pulsar timing + others

→ features in stellar streams, wakes in stellar density, lensing, etc.
[e.g. Calberg+, Erkal+, Belokurov+, Bushmann+, Ezaveh+, Penarrubia+, Feldmann+, Sandford+, Van Tilburg+, Dror+, etc.]

[NB1: DM clustering also impacts microlensing limits on PBHs]
[NB2: different DM scenarios imply different clustering properties]
Modeling Galactic subhalos

Theoretical framework well defined:

* Inflation model → **primordial power spectrum** (model dependent)
* DM-baryons coupling properties (model dependent)
* **Matter power spectrum** (model-dependent cutoff)
* Press-Schechter and extensions → **sub/halo mass function** (z)
...

...
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* Press-Schechter and extensions → **sub/halo mass function** ($z$)
  ...
* Fully **non-linear regime** with **cosmological simulations**
  => Statistical properties of sub/halos + links with cosmology
  ...

Via Lactea II, Diemand+08
Aquarius, Springel+08
Modeling Galactic subhalos

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* Press-Schechter and extensions → sub/halo mass function \((z)\)
* Fully **non-linear regime** with cosmological simulations
  \(\Rightarrow\) Statistical properties of sub/halos + links with cosmology
  
* Impact of baryons from hydro-runs / adiabatic growth of disks

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Via Lactea II, Diemand+08

Aquarius + baryons, Yurin+15

[see also Molitor+’15]

Aquarius, Springel+08

Eris, Guedes+11

Aquarius + baryons, Yurin+15
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... 

* Fully **non-linear regime** with **cosmological simulations**

\[ \rightarrow \text{Statistical properties of sub/halos} + \text{links with cosmology} \]

... 

* Impact of baryons from hydro-runs / adiabatic growth of disks

---

**PROBLEMS ARE**

* **Resolution limit**: compare \( 10^5 \text{M}_{\odot} \) with \( 10^{-10} \text{M}_{\odot} \) (in DM-only)
* ... getting worst in hydro-runs

...

* (Large uncertainties in baryonic physics)

...

* Modifications in cosmological inputs very expensive

...

* How is “Milky Way-like” defined?
* What’s special with “8 kpc” in a cosmological simulation? .... etc.
Making predictions for DM searches?

**The Milky Way a strongly constrained system!**
(specific history + properties + observational data)

[F. Iocco’s talk]
Making predictions for DM searches?

**The Milky Way a strongly constrained system!**
(specific history + properties + observational data)

[F. Iocco’s talk]

Cannot be a mere rescaling!

MW terminal velocities, McMillan ’11

Gaia: Data Release 2 (DR2) @ESA
Analytical model: defining the whole subhalo “phase space”

At MW formation, all (cosmological) properties factorize out

\[
\frac{d^n N^0}{d\omega^n} = N_0 \frac{d\mathcal{P}^0_V(\vec{x})}{dV} \times \frac{d\mathcal{P}^0_m(m)}{dm} \times \frac{d\mathcal{P}^0_c(c, m)}{dc}
\]
Analytical model: defining the whole subhalo “phase space”

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\frac{d^n \tilde{N}}{d\omega^n} = \tilde{N}_{\text{tot}} \frac{d\tilde{P}_V(\vec{x})}{dV} \times \frac{d\tilde{P}_m(m, \vec{x})}{dm} \times \frac{d\tilde{P}_c(c, m, \vec{x})}{dc}
\]

Step 1: compute tides induced by final MW halo

\[\Rightarrow\] parameter space becomes intricate!

\[\Rightarrow\] generic enough to be calibrated from simulations

\[\Rightarrow\] subhalo mass fraction \(\sim 10\%\) in range \((10^{-5}, 10^{-2}) M_h\)

(eg Diemand+08) fixes \(N_0\)
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\[
\frac{d^n \tilde{N}}{d\omega^n} = \frac{\tilde{N}_{tot}}{\tilde{K}_w} \frac{d\tilde{P}_V (\vec{x})}{dV} \times \frac{d\tilde{P}_m (m, \vec{x})}{dm} \times \frac{d\tilde{P}_c (c, m, \vec{x})}{dc}
\]

**Step 2:** compute tides induced by MW baryons

⇒ parameter space even more intricate

⇒ CANNOT be calibrated from simulations

\[
\frac{d^n N}{d\omega^n} = \frac{N_{tot}}{K_w} \frac{dP_V (\vec{x})}{dV} \times \frac{dP_m (m, \vec{x})}{dm} \times \frac{dP_c (c, m, \vec{x})}{dc}
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**Step 1:** compute tides induced by final MW halo
⇒ parameter space becomes intricate!

⇒ generic enough to be calibrated from simulations
⇒ subhalo mass fraction \( \sim 10\% \) in range \((10^{-5}, 10^{-2}) M_h\)
(eg Diemand+08) fixes \( N_0 \)

**Hard sphere argument:**
Subhalos track the evolving DM distribution, even after disruption.

⇒ redistribution of DM from subhalos to the smooth component.

⇒ only the final mass function knows about tidal stripping and disruption

\[
\frac{d^n \bar{N}}{d\omega^n} = \frac{\bar{N}_{\text{tot}}}{K_w} \frac{d\bar{P}_V(\vec{x})}{dV} \times \frac{d\bar{P}_m(m, \vec{x})}{dm} \times \frac{d\bar{P}_c(c, m, \vec{x})}{dc}
\]

**Step 2:** compute tides induced by MW baryons
⇒ parameter space even more intricate

⇒ CANNOT be calibrated from simulations
Input parameters \((m_{200}, r_{200}, c_{200})\) are not physical observables!

\[
(m_{200}, r_{200}, c_{200}) + \text{inner profile}
\]
→ set initial properties (flat background)
→ help fix scale parameters \(r_s\) and \(\rho_s\)

Physical parameters are
→ scale parameters \(r_s\) and \(\rho_s\)
→ tidal mass \(m_t\) and extension \(r_t\) + position
\[
(m_t, r_t < m_{200}, r_{200})
\]

\[
\rho_{\text{tot}}(R) = \rho_{\text{sm}}(R) + \rho_{\text{sub}}(R)
\]

Kinematic constraints [use McMillan’18 here]

Predicted [our model]
Setting the subhalo cutoff mass scale (thermal DM)

More details in Gaétan Facchinetti’s poster

Production/annihilation $\rightarrow$ chemical+thermal equilibrium
$\rightarrow$ Chemical decoupling $\rightarrow$ freeze out ($x_f=m/T_f\sim20$)
$\rightarrow$ Relic abundance fixed
NB: links with indirect searches

Elastic collisions $\rightarrow$ thermal contact with relativistic plasma after freeze out
Thermal contact ceases
$\rightarrow$ kinetic decoupling $\rightarrow$ free streaming ($x_k=m/T_k\sim10^2-10^4$)

Matter-radiation eq. $\rightarrow$ DM grows density fluctuations larger than free streaming scale
$\rightarrow$ Sets the minimal scale of DM halo
NB: links with direct searches / interaction with stars

Solve moments of Liouville-Boltzmann equation for coupled species

\[
\frac{d f(x^\mu, p^\mu)}{d\lambda} = \mathcal{C}[f]
\]

\[\Gamma_{\text{ann}} = n_\chi \langle \sigma_{\text{ann}} v \rangle\]

\[\Gamma_{\text{scatt.}} = n_f \langle \sigma_{\text{scatt}} v \rangle\]
Minimal halo mass from $\sim 10^{-12} M_{\odot}$ (>1 TeV WIMPs) to $\sim 10^{-3} M_{\odot}$ (<10 GeV WIMPs) 
Like relic abundance, fixed by interaction properties of DM particles!

[see also Schwartz+, Hofmann+, Green+, Bringmann+, Boehm+, Gondolo+, etc.]
Entangling the subhalo “phase space”: step 1

At MW formation, all (cosmological) properties factorize out

\[
\frac{d^nN^0}{d\omega^n} = N_0 \frac{d\mathcal{P}_V^0(\vec{x})}{dV} \times \frac{d\mathcal{P}_m^0(m)}{dm} \times \frac{d\mathcal{P}_c^0(c, m)}{dc}
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Step 1: compute tides induced by final MW halo

\[
\frac{d^n\tilde{N}}{d\omega^n} = \frac{\tilde{N}_{tot}}{K_w} \frac{d\tilde{\mathcal{P}}_V(\vec{x})}{dV} \times \frac{d\tilde{\mathcal{P}}_m(m, \vec{x})}{dm} \times \frac{d\tilde{\mathcal{P}}_c(c, m, \vec{x})}{dc}
\]

Step 2: compute tides induced by MW baryons

\[
\frac{d^nN}{d\omega^n} = \frac{N_{tot}}{K_w} \frac{d\mathcal{P}_V(\vec{x})}{dV} \times \frac{d\mathcal{P}_m(m, \vec{x})}{dm} \times \frac{d\mathcal{P}_c(c, m, \vec{x})}{dc}
\]
Global tidal effects

Solve EoM for test particle orbiting objects $m$ and $M$ ($m \ll M$) in co-rotating frame of frequency $\omega$ (King '62, Spitzer '87).

+ Demand force to vanish (Lagrange points L2, L3)

$$\ddot{x} = \frac{Gm}{x^2} - \frac{GM}{(R-x)^2} - \omega^2 \{ (\mu/m)R - x \} = 0$$

Point-like Jacobi tidal radius

$$r_{t,\bullet} = r_{t,\bullet}(R, m, M) = \left\{ \frac{mt}{3M} \right\}^{1/3} R$$

Extension to smooth systems

$$r_t = \left\{ \frac{m(r_t)}{3M(R) \left( 1 - \frac{1}{3} \frac{d \ln M(R)}{d \ln R} \right)} \right\}^{1/3} R$$

Smooth Jacobi tidal radius

Competition between global MW potential and internal subhalo potential → tidal radius
Entangling the subhalo “phase space”: step 2

At MW formation, all (cosmological) properties factorize out

\[
\frac{d^n N^0}{d\omega^n} = N_0 \frac{d\mathcal{P}_V^0(\vec{x})}{dV} \times \frac{d\mathcal{P}_m^0(m)}{dm} \times \frac{d\mathcal{P}_c^0(c, m)}{dc}
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Step 1: compute tides induced by final MW halo

Step 2: compute tides induced by MW baryons

\[
\frac{d^n \tilde{N}}{d\omega^n} = \frac{\tilde{N}_{\text{tot}}}{K_w} \frac{d\tilde{\mathcal{P}}_V(\vec{x})}{dV} \times \frac{d\tilde{\mathcal{P}}_m(m, \vec{x})}{dm} \times \frac{d\tilde{\mathcal{P}}_c(c, m, \vec{x})}{dc}
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\[
\frac{d^n N}{d\omega^n} = \frac{N_{\text{tot}}}{K_w} \frac{d\mathcal{P}_V(\vec{x})}{dV} \times \frac{d\mathcal{P}_m(m, \vec{x})}{dm} \times \frac{d\mathcal{P}_c(c, m, \vec{x})}{dc}
\]
Tides from stellar encounters and disk shocking

**Encounters with stars:**
[Spitzer+, Gerhard+, Carr+, Zhao+, Green+, Gnedin+, Berezinsky+, etc.]
* impulse approximation during fly-by
=> strongly in the very inner parts of MW

\[ \Delta E = \frac{1}{2} \int d^3r \rho_{\text{int}}(r)(\delta v_x - \delta v_y)^2 \]
\[ \Delta E = \frac{2\pi}{3} \left( \frac{2G_NM_*}{v_{\text{rel}}l^2} \right)^2 \int_0^R dr r^4 \rho_{\text{int}}(r) \]

**Disk shocking:**
[Ostriker+, Weinberg+, Gnedin+, Berezinsky+, etc.]
* impulse approximation during crossing
* adiabatic invariance correction
=> always strong

\[ \frac{dv_z}{dt} = g_d(R, z_p) - g_d(R, z_c) \]
\[ \simeq \Delta z \frac{\partial g_d}{\partial z} (z_c) \]
\[ \Delta v_z = \int dt \Delta z(t) \frac{\partial g_d}{\partial z} [z_c(t)] \]

**Tidal radius definition**
[demand \( E(r) < 0 \) after \( N \) crossings]
\[ r_{t,i} \text{ such that } \langle \epsilon_k \rangle (r_{t,i}) = -\tilde{\phi}(r_{t,i}, r_{t,i-1}) \]

\[ \epsilon_k(z) \equiv \frac{2 \frac{g_{z,\text{disk}}(z = 0)}{V_z^2} z^2}{A(\eta)} \]
Tidal disruption criterion (criteria?)

Subhalo tidal mass

\[ m_t = m(r_t) = 4\pi r_s^3 \int_0^{x_t} dx \, x^2 \rho(x, r_s) \, \zeta(x_t) \]

\[ dm = m_{200} - m_t \text{ given back to the smooth component} \]

Disruption function

\[ \zeta \left( x_t \equiv \frac{r_t}{r_s} \right) \equiv \theta(x_t - \varepsilon_t) \]

Disruption parameter \( \varepsilon_t \)

\[ x_t = \frac{r_t}{r_s} \geq \varepsilon_t \iff c_{200} \geq c_{\min}(R) \]

If circular orbit assumed,
Minimal concentration independent from mass!

From past numerical studies
\[ \varepsilon_t \approx 1 \]

Hayashi+03
Tidal disruption criterion (criteria?)

Subhalo tidal mass

\[ m_t = m(r_t) = 4\pi r_s^3 \int_0^{x_t} dx x^2 \rho(x r_s) \zeta(x_t) \]

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If circular orbit assumed,
Minimal concentration independent from mass!

**BUT ...**

If mini-cores dense enough, fast orbits should be resilient down to \( x_t << 1 \) \ldots (adiabatic invariance)

van den Bosh+’17’18 \( \Rightarrow \) tidal disruption strongly overestimated in simulations (resolution + softening issue). Also Errani+17.

\[ \Rightarrow \varepsilon_t << 1 \text{ most likely} \]

From past numerical studies \( \varepsilon_t \sim 1 \)
Impact of tidal disruption on number/mass density profiles

Constrained Galactic mass model from McMillan’18 assumed [NFW+bulge+gas/stellar thin/thick disks]

Sizable number density of tiny clumps expected locally! (~µpc size)
But on average, contribute a tiny fraction of the local density (~1 %)

Hidden above but important:
Mass + concentration pdfs have become spatial-dependent!
Amplification of annihilation rate in the Milky Way

Stref PhD. th ’18

Minimal subhalo mass matters for $\alpha > 1.9$
(always in the central regions due to effective mass index => local fluctuations suppressed)
[see also Silk&Stebbins’93, Bergström’98, JL+07, etc.]

Annihilation profile + local/integrated boost, Stref+17

DM annihilation profile: [consistent with kinematics!]

Antiproton flux

Disruption parameter
- $\epsilon_1 = 1$
- $\epsilon_1 = 0.5$
- $\epsilon_1 = 10^{-1}$
- $\epsilon_1 = 10^{-2}$

$$\alpha_m = 2$$
$$m_{\text{min}} = 10^{-6} M_\odot$$

Line-of-sight integral (J-factor – gamma-rays)

Antiproton flux

$$\sigma_V = 3 \times 10^{-26} \text{ cm}^2 / \text{s}$$
$$m = 1 \text{ TeV}$$
propagation : K15

Stref PhD. th ’18
* Analytical models of subhalos complementary to cosmological simulations
  + no resolution limit and fast
  + can easily probe different cosmologies
  + can be made consistent with dynamical constraints (e.g. the MW)
  + can apply to any DM candidate
  - have to rely on simplifying assumptions (e.g. spherical symmetry)

* Other analytical models on the market:
  - Berezinsky+: fully analytical (even density profiles), include baryons – qualitative estimates
  - van den Bosch+, Ando+, Hiroshima+: accretion+stripping, mass function (z), no baryons – EG gamma-rays
  - etc.

* Milky Way a perfect place to probe DM properties on small scales!
  → a strongly constrained system (global potential + baryons)
  → theoretical + dynamical self-consistence of DM distribution very important (smooth+subhalo components)

* Montpellier model (Facchinetti, Lavalle, Stref et al.) predicts properties of MW subhalo population
  - includes tidal stripping from both DM + baryons
  - consistent with MW kinematic constraints
  - qualitatively consistent with simulations results in relevant mass range
  - predictions for a series of observables: gamma-rays, antimatter cosmic rays, etc.

* Perspectives
  - full evolution from dark ages
  - detailed investigation of subhalo interactions with stars (DM capture)
  - application to PBHs
Backup
The dark halo: smooth vs subhalo component

\[ \rho_{\text{tot}}(R) = \rho_{\text{sm}}(R) + \rho_{\text{sub}}(R) \]

Overall profile constrained by non-linear theory: NFW, Einasto +/- cores

++++

**** Strongly constrained by MW kinematic data ****

\[ \rho_{\text{sub}}(R) = \frac{N_{\text{sub}}}{K_w} \frac{dP_V(R)}{dV} \int_{m_{\text{min}}}^{m_{\text{max}}} \int_{c_{\text{min}}(R)}^{c_{\text{max}}} dm \, dc \, m_t(r_t(c, m, R), m, c) \frac{dP_m}{dm} \frac{dP_c}{dc} \]

Strongly constrained by MW kinematic data

Series of kinematic constraints on baryons+DM mass models

++ will improve with Gaia ++

Density profiles for DM and baryons
From McMillan’11-'17
Stref PhD th.’18

Galactic components
- dark matter
- bulge
- disc (stars + gas)
**Tidal disruption criterion (criteria?)**

Subhalo tidal mass

\[ m_t = m(r_t) = 4 \pi r_s^3 \int_0^{x_t} dx \, x^2 \rho(x, r_s) \, \zeta(x_t) \]

\[ dm = m_{200} - m_t \] given back to the smooth component

Disruption function

\[ \zeta \left( \frac{x_t}{r_s} \right) \equiv \theta (x_t - \varepsilon_t) \]

Disruption free parameter \( \varepsilon_t \),

\[ x_t = \frac{r_t}{r_s} \geq \varepsilon_t \iff c_{200} \geq c_{\text{min}}(R) \]

Minimal concentration independent from mass!

But …

What about adiabatic invariants?

→ If mini-cores dense enough, fast orbits should be resilient down to \( x_t \ll 1 \) …

Recent work by van den Bosh+’17’18 suggests tidal disruption strongly overestimated in simulations. See also Errani+17.

NB: again a resolution issue → analytical arguments may catch on.

Minimal concentration vs position, Stref PhD th. ’18 ⇒ mean concentration gets spatial-dependent
(see also Pieri+11, Moline+15)
Post-tides properties

Concentration function cut from the left => spatial-dependent mass index!

Effective local mass index steeper than 2!

Modified local mass function, Stref+17
Evolution of species in the Early Universe

\[
\frac{df(x^\mu, p^\mu)}{d\lambda} = \hat{C}[f]
\]

\[
\frac{dY_x}{dx} \propto -\frac{g_x^{1/2}(x)}{x^2} \langle \sigma v \rangle \left\{ Y_x^2 - Y_{eq}^2 \right\}
\]

\[T_x \equiv \left\langle \frac{p^2}{3m_x} \right\rangle = \frac{g_x}{3m_x n_x} \int p^2 f_x(p, t) \frac{d^3p}{(2\pi)^3}.
\]

\[
\frac{\partial T_x}{\partial t} + 2HT_x = \gamma(T)(T - T_x)
\]

\[
\gamma(T) = \frac{1}{48g_x m_x^3 \pi^3} \sum_{\text{species } i} \int_{m_i}^{\infty} d\omega f_i^{eq}(\omega, t) \frac{\partial}{\partial \omega} \left( \int_{-4p_{cm}}^{0} (-t) |\hat{M}_i|^2 dt \right)
\]

\[
\frac{d \ln(y_x)}{d \ln(x_x)} = - \left( 1 + \frac{d \ln(h_{\text{eff}}(T))}{3d \ln(T)} \right) \frac{\gamma(T)}{H} \left( 1 - \frac{y_x^{eq}}{y_x} \right)
\]
Closest visible object

SL17, $\epsilon_t = 10^{-2}$

![Graph showing the distribution of DM around the Earth with different models and parameters, including DM only, Phat-ELVIS, SL17 with $\epsilon_t = 10^{-2}$, SL17 with $\epsilon_t = 1$, $J_{0.5^\circ}$, and $J_{full}$, and highlighting the annihilation region.](image_url)
Subhalo eccentricity distribution

Facchinetti+, in prep
Boost factors in context

Bergström’09

Boost factor depends on integration volume!

See also Silk & Stebbins’93, Begström+99, Lavalle+07-08
$J$ factors! (at last)

$\alpha_m = 1.9$

$m_{\text{min}} = 10^{-6} M_\odot$

Disruption parameter
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- $\epsilon_t = 0.5$
- $\epsilon_t = 10^{-1}$
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$\alpha_m = 2$

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Disruption parameter
- $\epsilon_t = 1$
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Stref PhD th ’18
Kinetic decoupling, free streaming scale, and small-scale structures

\[
\lambda_{fs} = a_{eq} \int_{t_{kd}}^{t_{eq}} dt \frac{v(t)}{a(t)} \approx v_{kd} \frac{a_{kd}}{a_{eq}} / H_{eq}
\]
Searches for thermal dark matter

\[ \Gamma_{\text{ann}} = n_X \langle \sigma_{\text{ann}} v \rangle \]

\[ \Gamma_{\text{scatt.}} = n_f \langle \sigma_{\text{scatt}} v \rangle \]

- **Production** at colliders (model dependent)
  - => collider searches

- **Annihilation/decay** rate potentially large in dense DM regions: centers of halos + CMB
  - => indirect searches

- **Beware velocity dependence**
  - (scalar exchange between fermions v-suppressed; pseudo-scalar exchange is not)

- **Elastic or inelastic scattering**
  - nuclear recoils at underground experiments
  - => direct searches

  - scattering with astrophysical objects
  - => stellar physics
  - => neutrinos from capture+annihilation in stars
  - => indirect searches

- **Beware velocity dependence**
  - (pseudo-scalar exchange v-suppressed; scalar exchange is not)
Tides from stellar encounters and disk shocking

Encounters with stars:
(Ostriker+, Weinberg+, Gnedin+, 80-00, Berezinsky+03)
* impulse approximation during fly-by
=> strong in the very inner parts of MW

\[
\Delta E = \frac{1}{2} \int d^3r \rho_{\text{int}}(r)(\delta v_x - \delta \bar{v}_x)^2
\]

\[
\Delta E = \frac{2\pi}{3} \left( \frac{2G_N M_\star}{v_{\text{rel}} l^2} \right)^2 \int_0^R dr \ r^4 \rho_{\text{int}}(r)
\]

Disk shocking:
* impulse approximation during crossing
* adiabatic invariance correction
=> always strong

\[
\frac{dv_z}{dt} = \left( g_d(R, z_p) - g_d(R, z_c) \right)
\simeq \Delta z \ \frac{\partial g_d}{\partial z}(z_c),
\]

\[
\Delta v_z = \int dt \ \Delta z(t) \ \frac{\partial g_d}{\partial z}(z_c(t))
\]

\[
\epsilon_k(z) \equiv \frac{2 g_{z, \text{disk}}^2(z = 0) z^2}{V_z^2} \ A(\eta)
\]

Tidal radius definition
[demand \( E(r) < 0 \)]

Differential definition (default)

\[ r_{t,i} \text{ such that } \langle \epsilon_k \rangle(r_{t,i}) = -\bar{\phi}(r_{t,i}, r_{t,i-1}) \]

Integrated definition

\[ r_t \text{ such that } N_{\text{cross}} E_k(r_t, R) = E_b(r_t) \]

Fit from D’Onghia+10

\[
\tilde{E}_k(r_t, R) = \frac{(1.84 \ r_1/2)^2 \ g_{z, \text{disk}}^2}{3 \ \bar{\sigma}_v^2 \ V_z^2}
\]

\[
E_b(r_t) = \frac{1.84 \ r_1/2 \ g_{z, \text{disk}}^2}{3 \ \bar{\sigma}_v^2 \ V_z^2}
\]