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Supermassive Black Holes from Quantum Condensate Dark Matter

- Black Hole/Dark Halo Ratio from Rotation and Axion -

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We explore the possibility that quantum condensed DM/DE formed SMBH before the star formation. The detail is in arxiv:1903.02986.

DE→DM→SMBH…dark species are connected with each other

1. Introduction

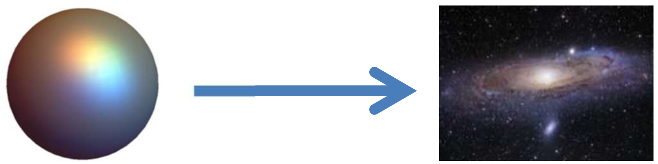
1. Why most of the galaxies have Supermassive BH (SMBH)?
   …$10^{6\text{–}10}M_{\odot}$
2. Why SMBH is located at the center of the galaxy?
3. Why SMBH is formed so early?… $z \approx 6 - 7.5$
4. Why SMBH and the galaxy bulge have universal correlation
   … $M_{\bullet} = \dfrac{f\kappa\sigma^4}{4\pi G^2} \propto \sigma^4$

SMBH seems to define the center of the galaxy

→ SMBH was formed first @$z \approx 10 - 20$

→ The SMBH triggered the star formation and the galaxy @$z \leq 10$

i.e. the coevolution might be rapid…



2. How SMBHs were formed?

If BH formed by Baryons $m_{pl}^{3} / m_{p}^{2} \approx M_{\odot}$ ($m_{pl} \equiv \sqrt{\hbar c / G}$)

➔ small BH of size $M_{\odot}$ coalesces → self-gravitating system

➔ coalescence requires too long time to form $10^{7-9} M_{\odot}$

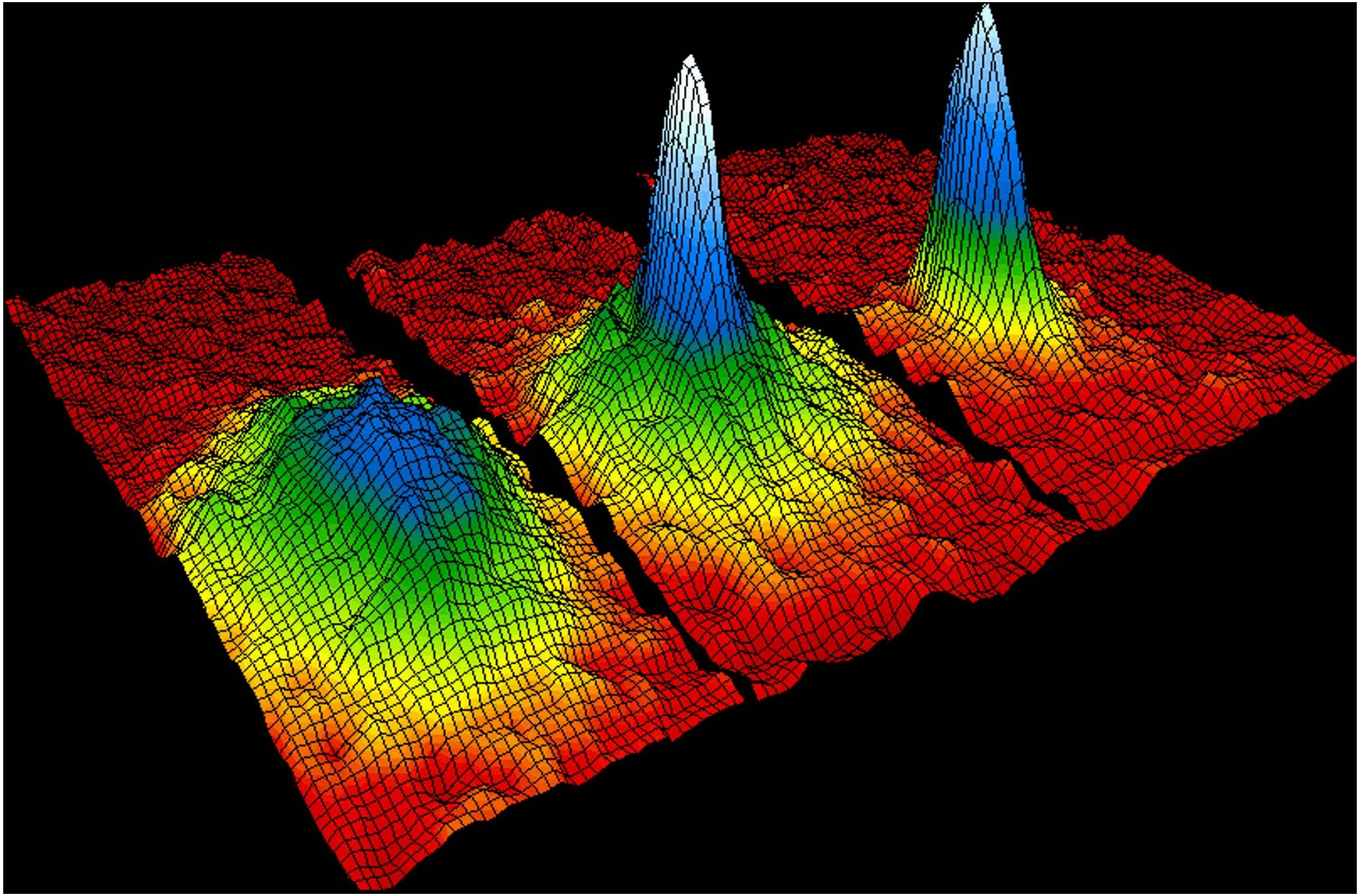
➔ Accretion also requires too long time

($10^{3} M_{\odot} \rightarrow 10^{9} M_{\odot}$ needs $6.2Gy$ by Edd. Acc.)

- The above assumed **particles**.
- We would like to consider **coherent wave** for rapid collapse**.**
- On the other hand, we once proposed the unified model of DE/DM (Fukuyama, MM 2009)

(**DE**=condensate, **DM**=gas *i.e.* **same boson but different phases**)

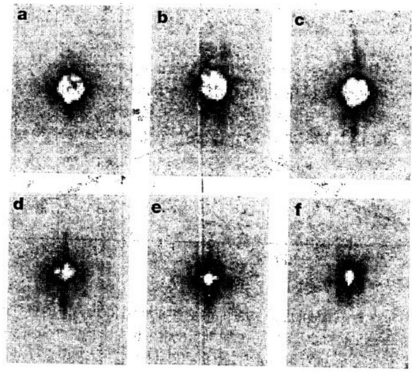
Our problem is… How BEC wave collapses to form SMBH.



Rb atom time series in phase space https://www.youtube.com/watch?v=1RpLOKqTcSk

Experimentally found BEC in dilute gases of alkali atoms:

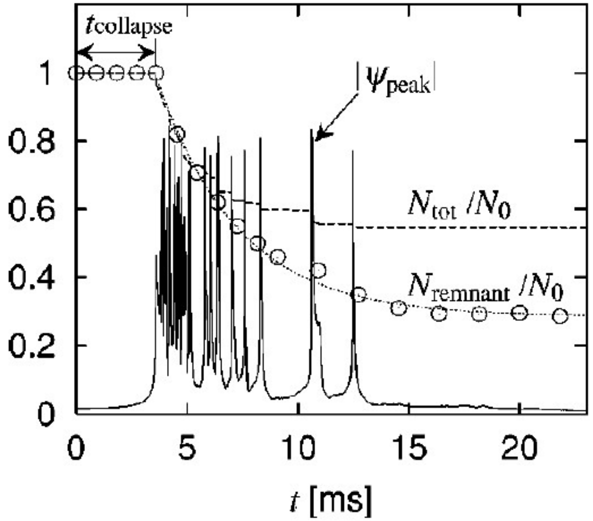
★ **Experiments** http://amo.phy.gasou.edu/bec.html/

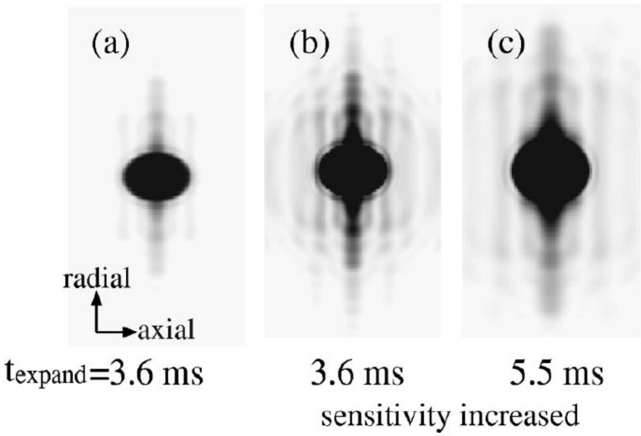


boson-nova **in BEC experiments: Wieman et.al.[1] [2]**

BEC actually collapses to 10^{-5} times denser after 5ms,and burst explosion.
BEC decay ejection of jet oscillation continues!

★ **Numerical work:**
Hiroki Saito
and Masahito Ueda
2002





[1] J.R. Anglin and W. Ketterle, Nature (London) 416 (2002 March 14) 211 for review.
J.M. Gerton, et al. (2000), S.L. Cornish et al. (2000) E.A. Donley et al. (2001).

[2] **H. Saito and M. Ueda, Phys. Rev. A 65 (2002) 033624.**

BEC condition:
(thermal de Broglie length) > (mean separation of particles):

$$\lambda_{dB} \equiv \left(\frac{2\pi\hbar^2}{mkT} \right)^{1/2} > r \equiv n^{-1/3}$$ **i.e.** $$kT < \frac{2\pi\hbar^2 n^{2/3}}{m}$$ **and**

Cosmic evolution: $n = n_0 \left(\frac{m}{2\pi\hbar^2} \frac{T}{T_0} \right)^{3/2}$ **has the <u>same</u> temperature dependence!**

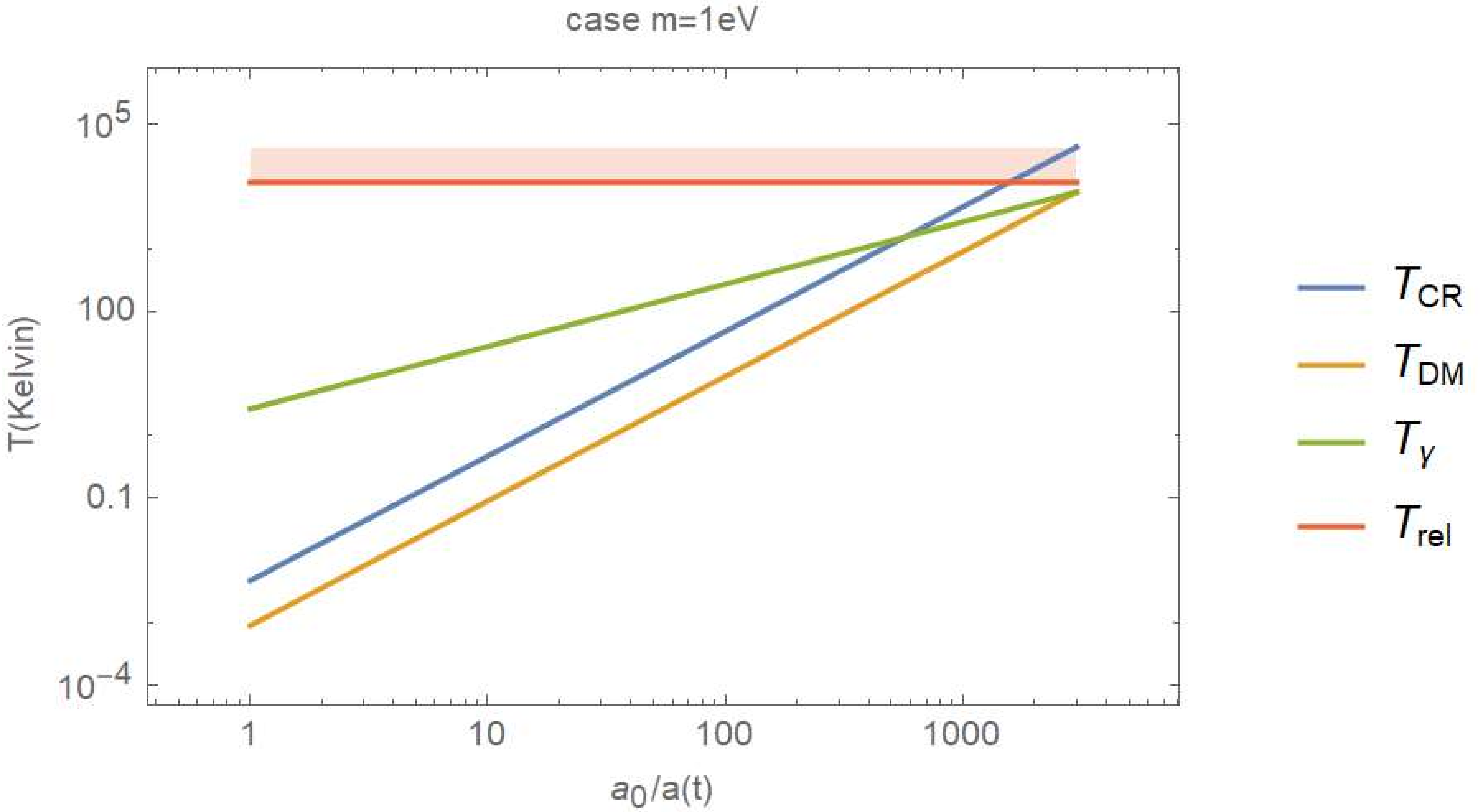
$$\Rightarrow \quad T_{cr} = 4 \times 10^{-3} \left(eV / m \right)^{5/3} K$$

$$T_{DMt} = T_{DM0} \left(a_0 / a_t \right)^2$$

$$T_{\gamma t} = T_{\gamma 0} \left(a_0 / a_t \right)$$

- Therefore, once BEC, it keeps BEC in the adiabatic process.

⇒ If $T_{DMt} = T_{\gamma t}$ before the decoupling, then DM is quantum condensed for $m < 10\text{eV}$.



- Kaup limiting mass $M_{kaup} = 0.633\frac{\hbar c}{Gm} \approx \frac{{m_{pl}}^2}{m}$ beyond which the BEC simply collapses.

- However, for non-adiabatic collapse, T increases and BEC melts into gas.

Ex) $T = 5.6 \times 10^{-2}(m\,/\,eV)K \quad for \quad M = 10^{12}M_{\odot},\, R = 10kpc$

…and exceeds the critical temperature.

Literature:

- Critical phenomena of BH: Choptuik 1993, Gundlach 2007

  $M_{BH} \propto (\,p - p_{*}\,)^{\gamma}$
- SMBH formation from BEC DM/DE

  Nishiyama et al. 2004. Fukuyama et al. 2006.
- Gupta and Thareja 2017, Shavanis 2017. Similar BEC collapse using Gaussian app.

3. SMBH formation

We solve the wave equation

$$i\hbar\frac{\partial\psi(t,\mathbf{x})}{\partial t}=\left(-\frac{\hbar^2}{2m}\Delta+m\phi+g|\psi|^2\right)\psi:$$ Gross Pitaevski eq. for BEC

with

$\Delta\phi=4\pi Gm|\psi|^2$: Poisson eq.

where $\psi(t,\mathbf{x})$ is the BEC condensate macro wave function

- Newtonian approximation…discarding the back reaction to space-time, may be a simple indicator of the BH formation
- Gaussian approximation…reduction of PDE to a set of ODE

$$\psi(t,x)=Ne^{-r^2/(2\sigma(t))^2+ir^2\alpha(t)},\ \phi(t,x)=-\mu(\tau)\,e^{-r^2/(2\tau(t))^2}$$

★ Isotropic collapse

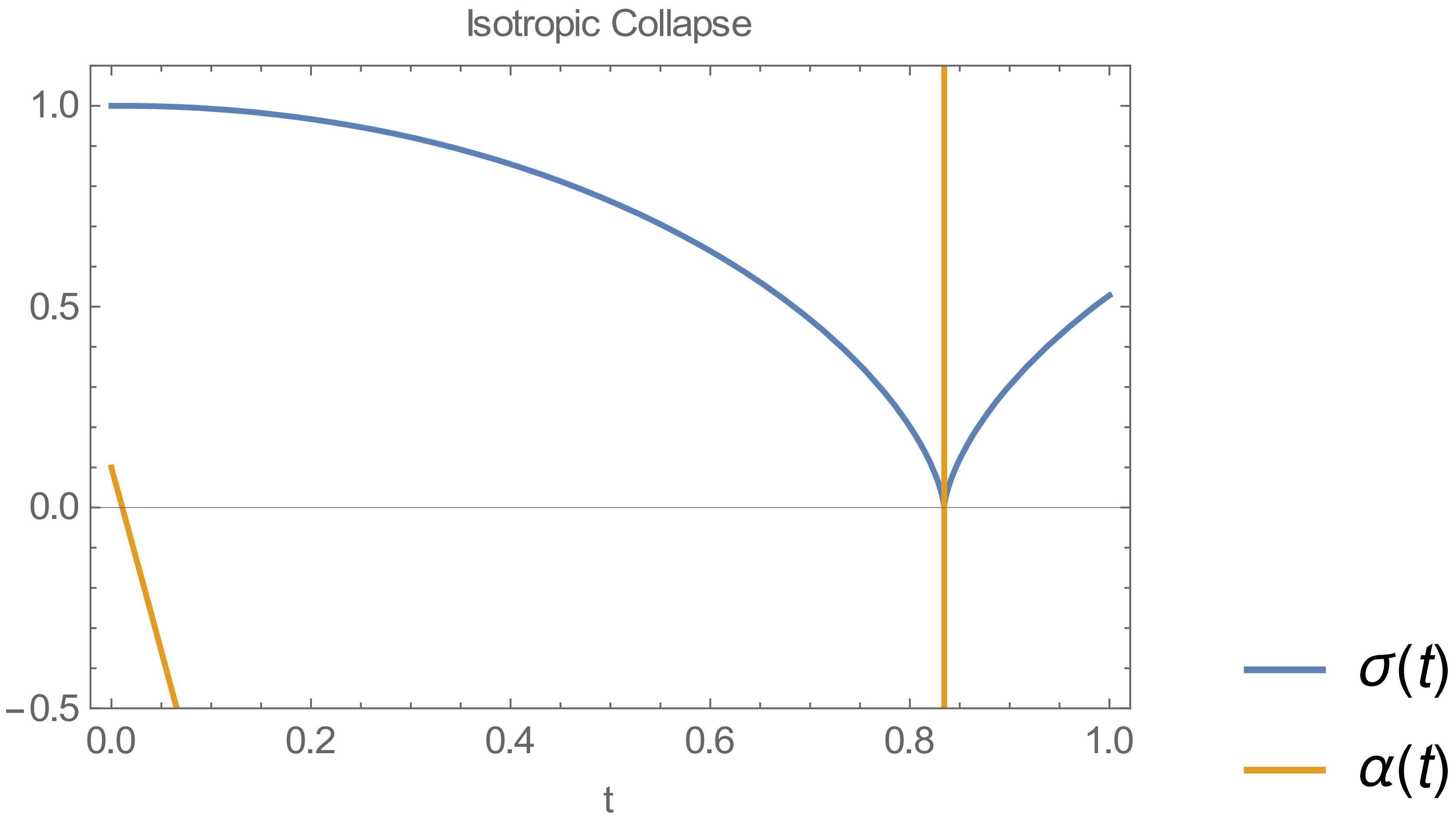
$$L = \quad (i\hbar/2)\left(\psi^\dagger\dot{\psi} - \dot{\psi}^\dagger\psi\right) - \left(\hbar^2/2m\right)\nabla\psi^\dagger\nabla\psi - (g/2)\left(\psi^\dagger\psi\right)^2$$
$$- (1/8\pi G)\nabla\phi\nabla\phi - m\phi\psi^\dagger\psi.$$

- Putting the Gaussian form of $\psi\left(t,\mathbf{x}\right)$ and $\phi(t,\mathbf{x})$ into the original Lagrangian.
- Then spatially integrate it to yield the effective action.
- Replace the dispersion $\tau(t)$ in the Laplace equation.
- Inserting the formal solution for $\alpha(t)$
- and we obtain the Effective Lagrangian

$\Rightarrow$

$$L_{\rm eff} = 1/16(-(2\sqrt{2}gN^2)/(\pi^{3/2}\sigma(t)^3) - (12N\hbar^2)/(m\sigma(t)^2) - (48N\hbar^2\alpha(t)^2\sigma(t)^2)/m$$
$$+ (32\sqrt{2}N\mu(t))/(\sigma(t)^2(2/\sigma(t)^2 + 1/\tau(t)^2)^{3/2}))$$
$$- (3\sqrt{\pi}\mu(t)^2\tau(t))/G - 24N\hbar\,\sigma(t)^2\alpha'(t)).$$

Time evolution of the dispersion $\sigma(t)$ and the phase $\alpha(t)$ of $\psi(t,\mathbf{x})$:



$\Rightarrow$ BH is easily formed if $r_S > \sigma(t)$ or equivalently $M > M_{\text{kaup}}$

How about the anisotropic case?

★ Anisotropic case

Putting $\psi(t,x) = \exp\left[i\mathrm{x}_1^2\alpha_1(t) - \frac{\mathrm{x}_1^2}{2\sigma_1(t)^2} + i\mathrm{x}_2^2\alpha_2(t) - \frac{\mathrm{x}_2^2}{2\sigma_2(t)^2} + i\mathrm{x}_3^2\alpha_3(t) - \frac{\mathrm{x}_3^2}{2\sigma_3(t)^2}\right]$

and $\phi(t,\mathbf{x})$ into the original Lagrangian, spatially integrate it.

- Remove the dispersion $\tau(t)$ of $\phi(t,\mathbf{x})$ by the Laplace equation
- Input the formal solution for $\alpha(t)$ into it, we get the effective Lagrangian:

$$L_{\text{eff}} = -\frac{N}{4m\sigma_1(t)^2} - \frac{N\hbar^2}{4m\sigma_2(t)^2} - \frac{N\hbar^2}{4m\sigma_3(t)^2} - \frac{gN^2}{4\sqrt{2}\pi^{3/2}\sigma_1(t)\sigma_2(t)\sigma_3(t)} + \ldots - \frac{1}{4}mN\sigma_3(t)\sigma_3''(t)$$

- The corresponding effective potential:

$$V_{\text{eff}} = \frac{gN}{2\sqrt{2}\pi^{3/2}\sigma_1(t)\sigma_2(t)\sigma_3(t)} + \frac{25\sqrt{\frac{10}{\pi}}GN\sqrt[3]{\sigma_2(t)}\sqrt[3]{\sigma_3(t)}}{243\sigma_1(t)^{5/3}} + \ldots + \frac{\hbar^2}{2m\sigma_1(t)^2} + \frac{\hbar^2}{2m\sigma_2(t)^2} + \frac{\hbar^2}{2m\sigma_3(t)^2}$$

Tine evolution of the dispersions:

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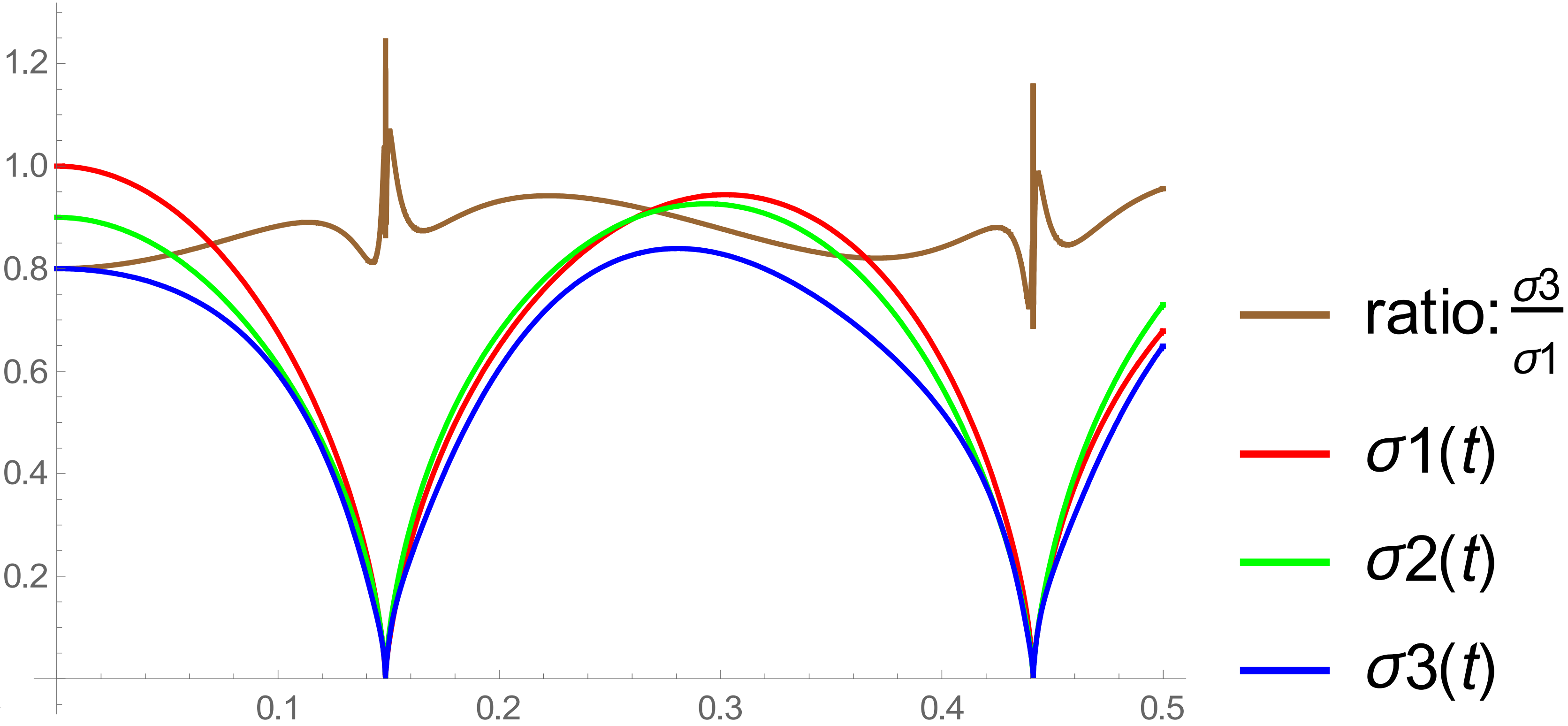

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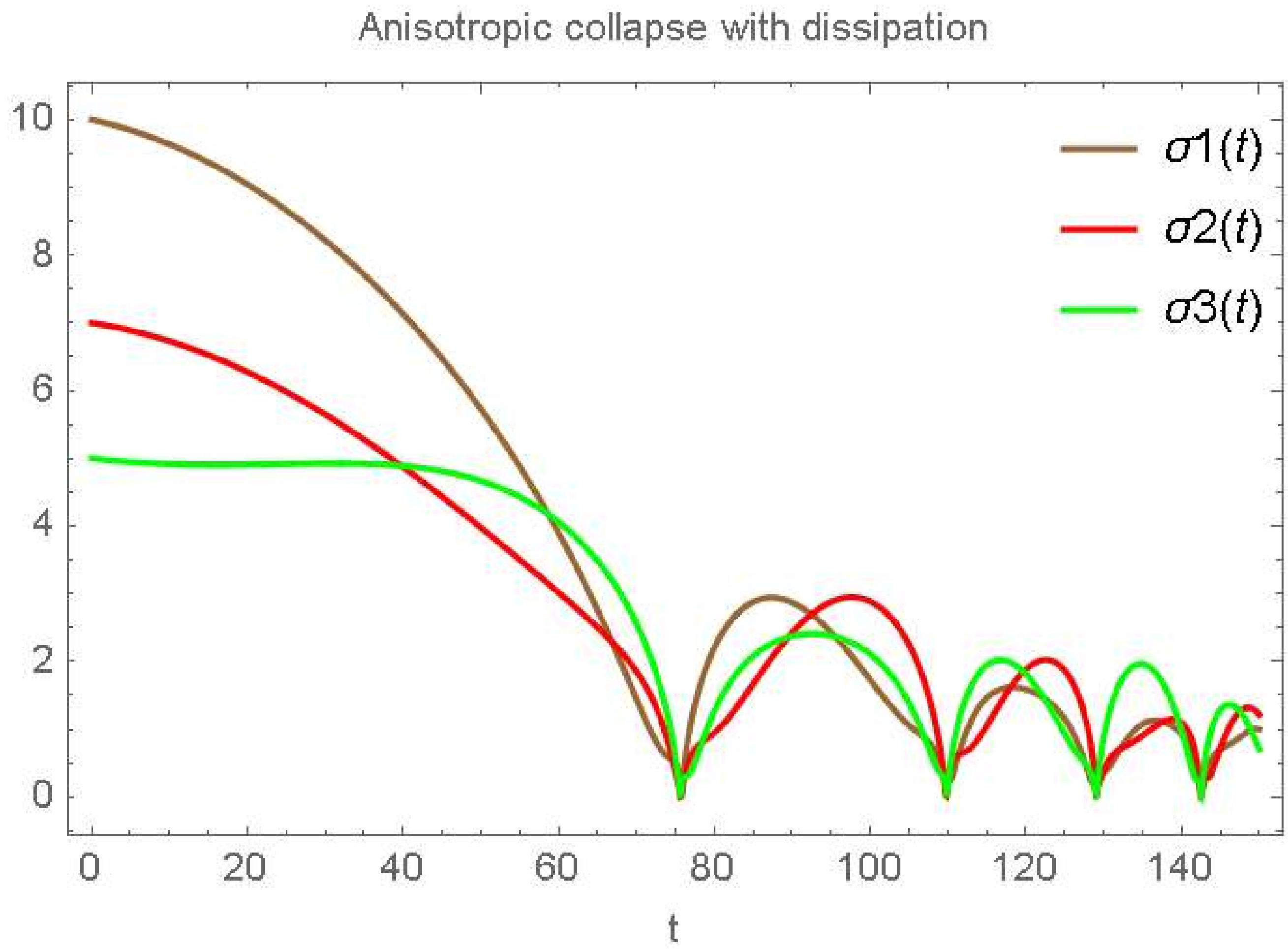
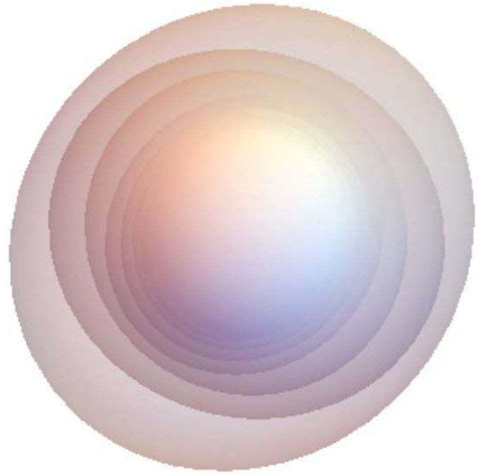
- Anisotropic BEC can collapse to form BH. Initial anisotropy is almost maintained but violently oscillate at BH formation.
- The bounce repeats almost periodically.

★ Dissipative collapse

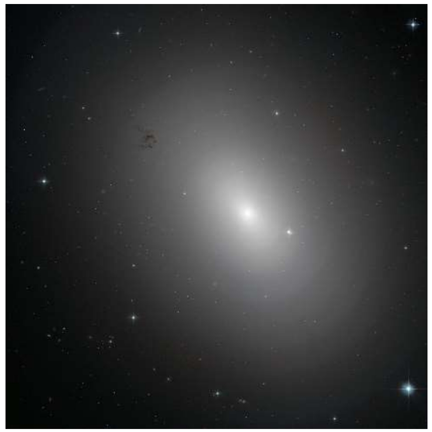
non-adiabatic case

$$L_{\rm diss} = e^{\gamma t} L$$





Forms concentric shell structure



cf. [NGC 3923] NASA

★ BEC with the angular momentum

For the wave function,

$$\psi(t,\mathbf{x}) = e^{-\frac{r^2}{2\sigma(t)^2}+ir^2\alpha(t)} Y_l^m(\theta,\phi)$$

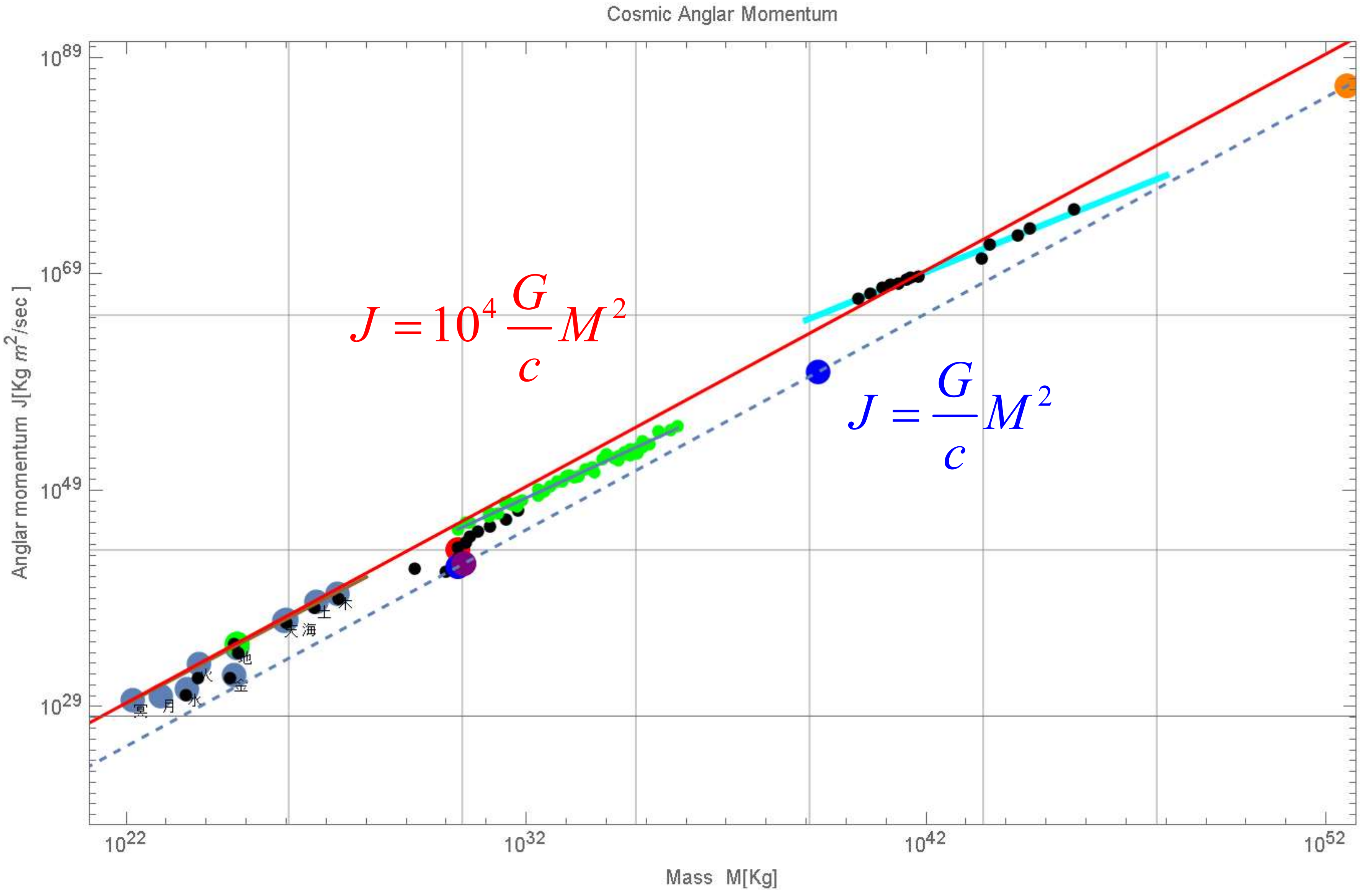
Effective potential becomes:

$$\frac{gN}{\sigma(t)^3} - \frac{Gm^2N}{\sigma(t)} + \frac{A^2}{m\sigma(t)^2}$$

⇒ Large angular momentum A prevents the BH formation

4. Angular momentum prevents the SMBH formation

◆ Cosmic angular momentum　Nakamichi et.al. (2009)



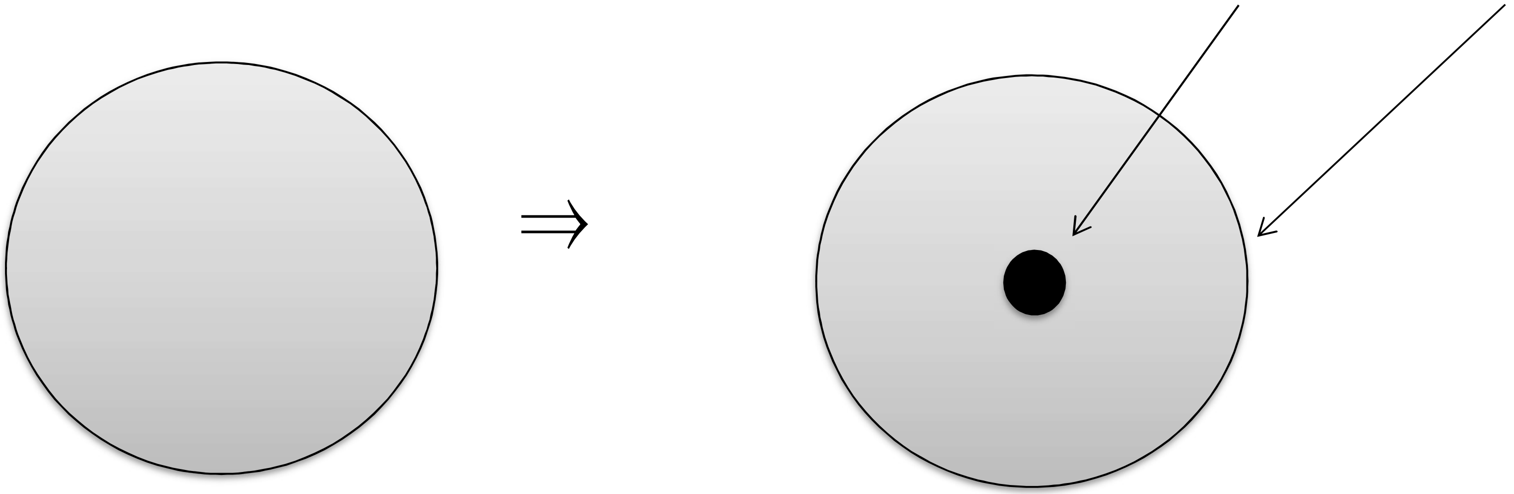
- Planets to the cluster of galaxies: $A = \alpha \frac{G}{c} M^2 = \alpha\hbar\left(\frac{M^2}{m_{pl}^2}\right) \quad \alpha = 10^4$

The balance $-\frac{Gm^2\mathrm{N}}{\sigma(t)} + \frac{\hbar^2 A^2}{m\,\sigma(t)^2}$ yields $M_{cr} \approx 100M \rightarrow$ <u>No collapse</u>

i.e. **Angular momentum prevents the SMBH formation as a whole.**

or…

Angular momentum should control the division of SMBH and DH.



Astronomical structures are protected by their angular momentum free from entirely being BH.

Solution 1: *...reduce the wall of angular momentum*

→ Go back to the early stage when the dark halo (DH) fully acquires the angular momentum

i.e. BH before tidal torque mechanism

→ Earlier the formation stage, larger the BH

(an example of the downsizing)

Solution 2: *...conquer the wall of angular momentum*

→ Introducing attractive interaction $g < 0$ such as Axion

5. SMBH formation controlled by angular momentum (solution 1)

Density profile: $\rho = \beta(t)\frac{\rho_0}{(\frac{r}{r_0})^2}$

Rigid rotation: $v = \alpha(t)\Omega r$

Angular momentum: $J(r) = \frac{4}{3}\pi\alpha(t)\beta(t)\rho_0 {r_0}^2 r^3 \Omega$

Mass: $M(r) = 4\pi\beta(t)\rho_0 {r_0}^2 r$

◆BH formation condition: $\mu \leq 1$, $\mu \equiv \frac{cJ}{GM^2} = \frac{cr\Omega(t)\alpha(t)}{12\pi\beta(t)G\rho_0 r_0{}^2}$

$\Omega(t)$: acquisition of the angular momentum $J \propto \alpha(t)\beta(t) \propto t$

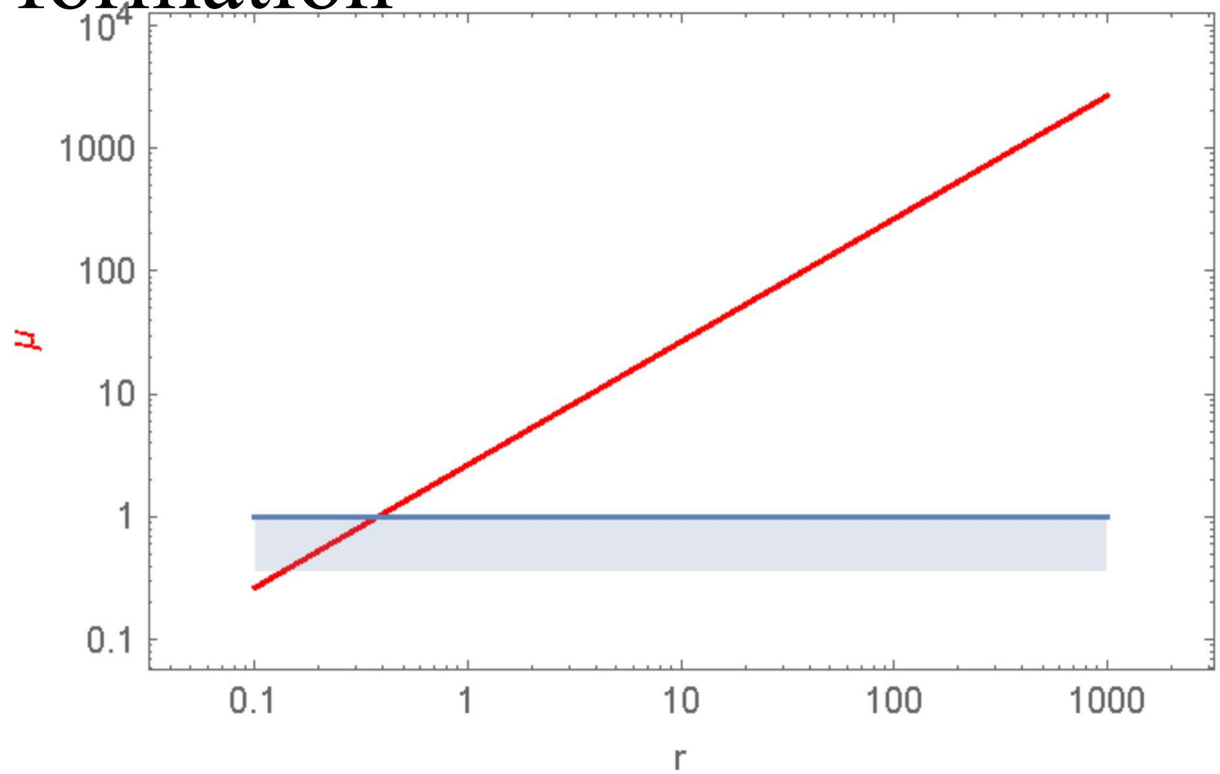
(TTT) ⇒**Prevents** BH formation

$\beta(t)$: development of the density fluctuations $M \propto \beta = \left(\frac{t}{3\ G\ \text{year}}\right)^{2/3}$

⇒**Promotes** BH formation

BH formation time scale:

$$t_{ff} = G^{-1/2}\left(\frac{M(r)}{\frac{4\pi}{3}r^3}\right)^{-1/2}$$

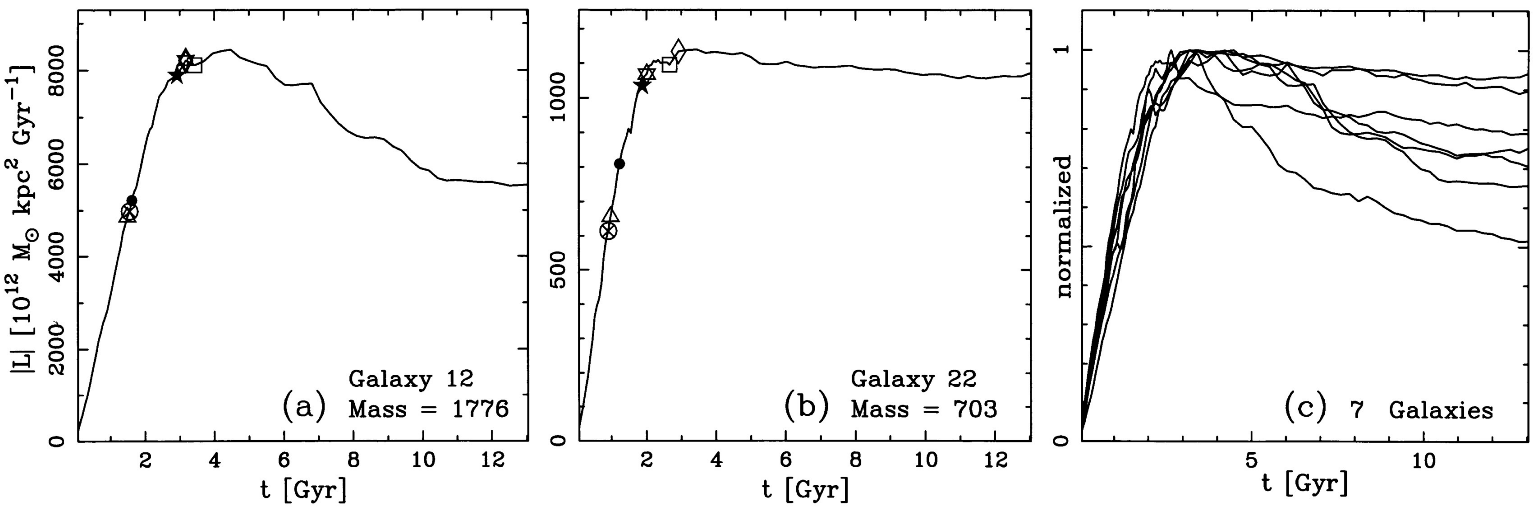


- Tidal Torque Theory (TTT)

$J(t) = \int_{a^3V} dr\, \rho r \times \dot{r} = \rho_b a^5 \int_V\ dx\,(1+\delta)x \times u$ yields

$J(t) = \rho_0 {a_0}^3 a(t)^2 \dot{D}(t) \int_V\ dq\, q \times \nabla\varphi(q) \propto t$ *i.e.* linear evolution

$(r = a(t)x, u = \dot{x}, x(q,t) = q - D(t)\nabla\varphi)$



Sugerman et.al. MNRAS311 762 (2000)

A typical galaxy acquires its present amount of angular momentum within about 3Gy…by numerical calculations.

Solving $\boxed{t_{ff}(t,r) = t \text{ and } \mu(t,r) = 1}$, we obtain

t = BH formation time scale: $\frac{2\sqrt{G}\sqrt{\mathrm{Mtot}}\sqrt{6\pi}}{c\sqrt{\mathrm{Rtot}}\Omega}$

r = BH region: $\frac{6G^{7/6}\mathrm{Mtot}^{7/6}(3\pi)^{1/6}}{c^{4/3}\mathrm{Rtot}^{7/6}{t_J}^{1/3}\Omega^{4/3}}$

M = BH mass: $\frac{12G^{3/2}\mathrm{Mtot}^{5/2}\sqrt{3\pi}}{c^2\mathrm{Rtot}^{5/2}t_J\Omega^2}$

Typically, for

$M_{tot} = 10^{11}\mathrm{M}_{\odot}$, $R_{tot} = 10\mathrm{kpc}$, $\Omega = \frac{200\frac{\mathrm{km}}{\mathrm{Sec}}}{R_{tot}}$, $t_J = 3\mathrm{gigayear}$,

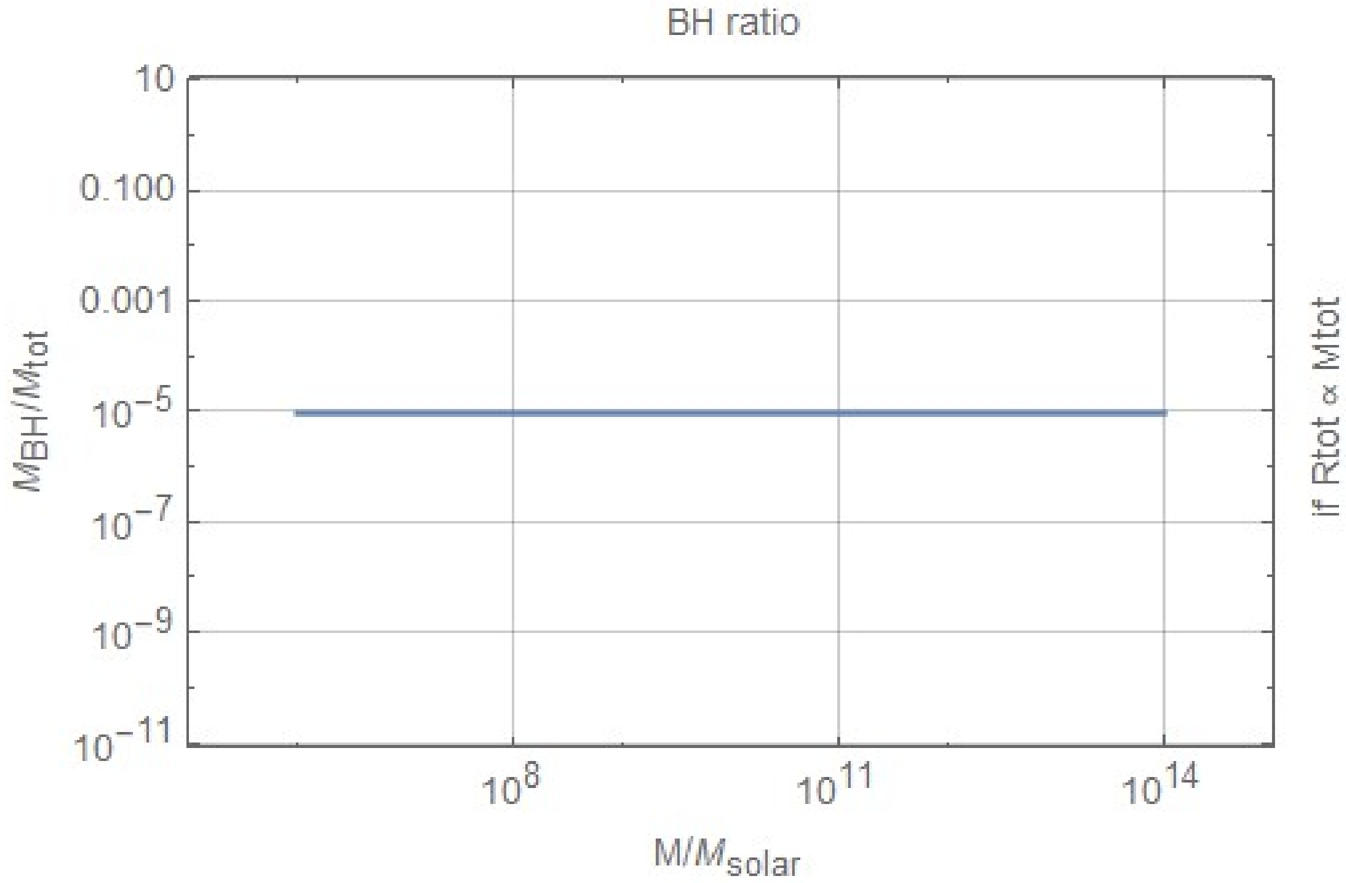
we have

$\mathrm{r} = 20\mathrm{pc}$, $\quad$ $\mathrm{t} = 0.9 * 10^6$ year

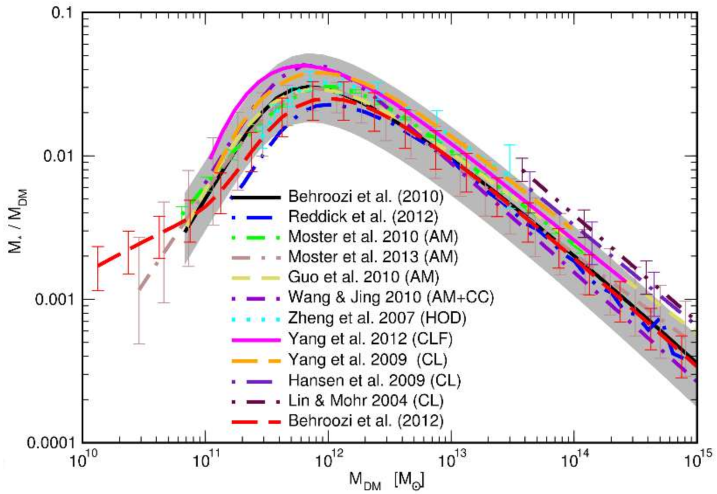
$\mathrm{M} = 0.94 * 10^7$ solarmass $\quad$ …

- More generally, BH/DH ratio(M_{BH}/M_{DH}) is given as

Assuming $M_{tot} \propto R_{tot}$



i.e. $\frac{M_{BH}}{M_{DH}} \approx 10^{-5}$

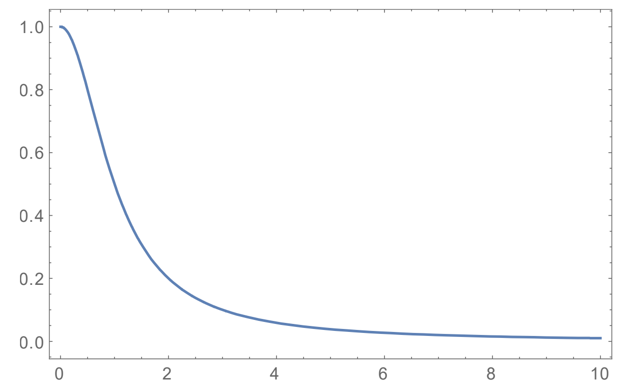


Observation: Kormendy & Ho (2013)

6. Axion case: angular momentum vs. attractive force (solution 2)

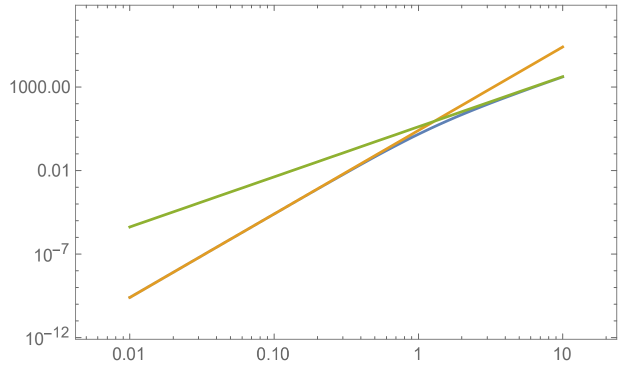
Density profile: $\rho = \frac{\rho_0}{1+(\frac{r}{r_0})^2}$

Rigid rotation: $v = r\Omega$



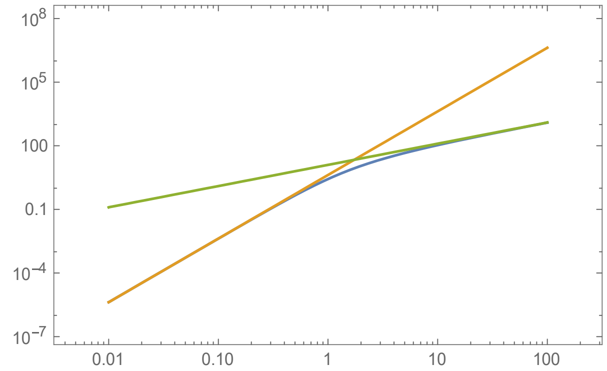
angular momentum

$$J(r) = 4\pi\rho_0\Omega r_0^{\,2}(\frac{1}{3}r^3 + r_0^{\,3}\tan^{-1}\left(\frac{r}{r_0}\right) - r r_0^{\,2})$$

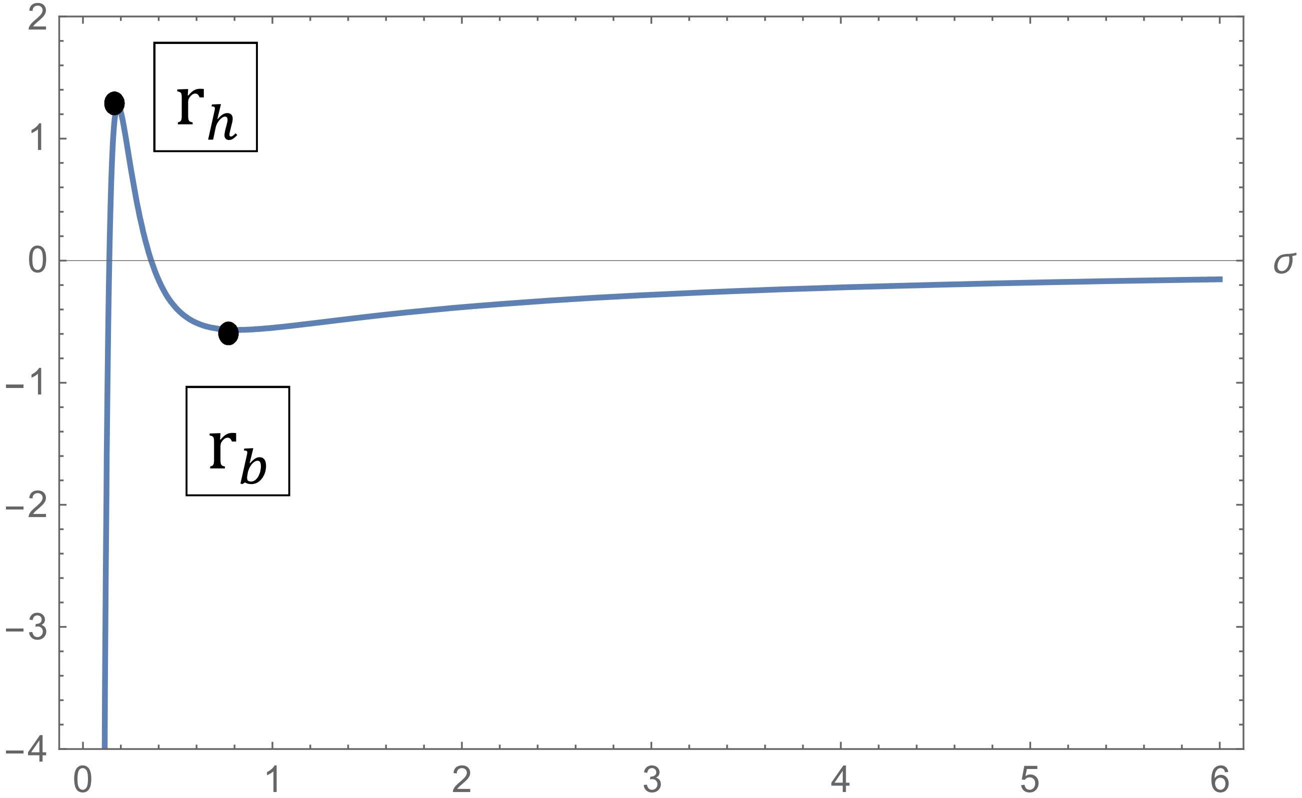


mass:

$$M(r) = 4\pi\rho_0 r_0^{\,2}(r - r_0\tan^{-1}(\frac{r}{r_0}))$$



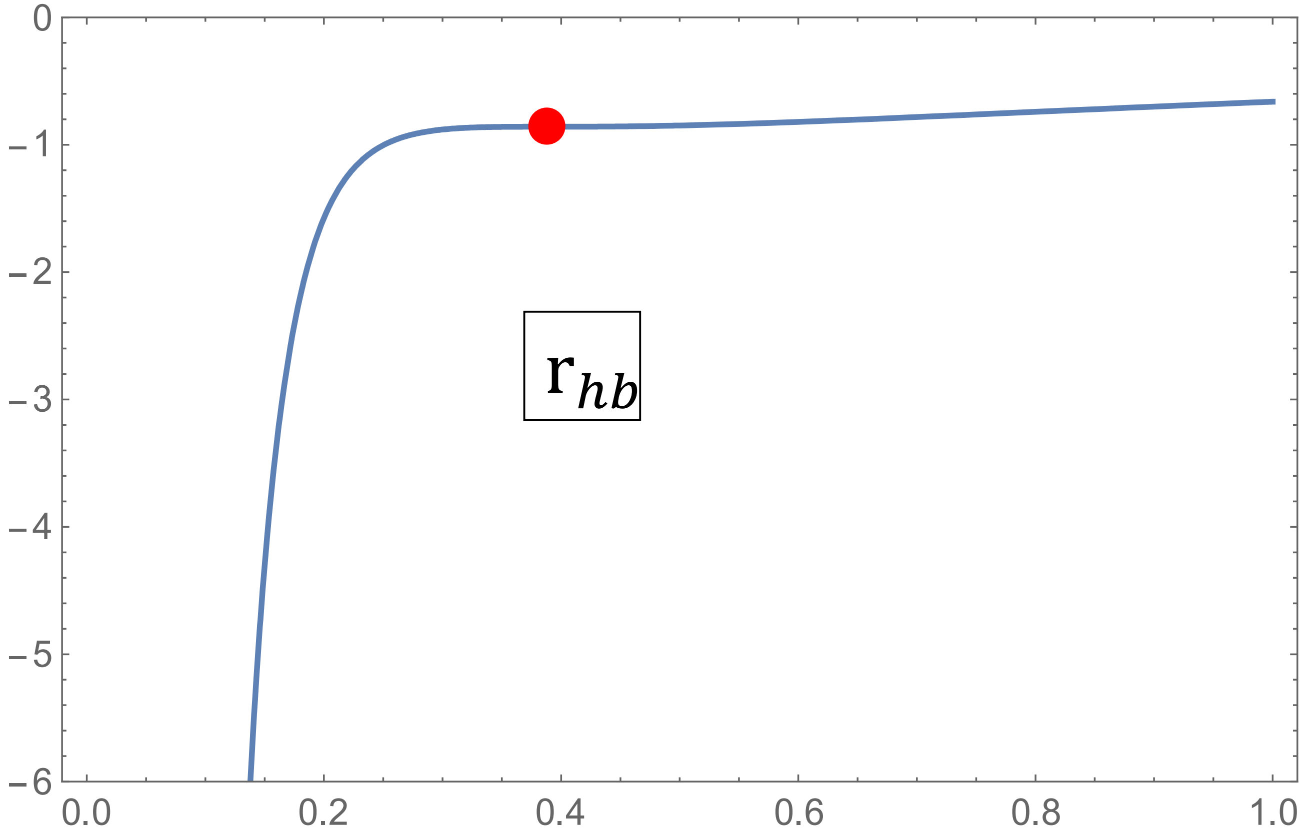
Effective potential: $V_{\text{eff}} = \frac{g\mathrm{M}}{\sigma^3} - \frac{G\mathrm{m}^2\mathrm{M}}{\sigma} + \frac{J^2}{\mathrm{m}\sigma^2}, \quad g = -\frac{4\pi a_s \hbar^2}{m}$



This potential profile prevents SMBH formation

$$\begin{Bmatrix} r_h = \dfrac{\mathrm{J}^2 m^3 - \sqrt{\mathrm{J}^4 m^6 - 48\pi a_s G m^3 \mathrm{M}^6 \hbar^2}}{2 G m^3 \mathrm{M}^3} \\ r_b = \dfrac{\mathrm{J}^2 m^3 + \sqrt{\mathrm{J}^4 m^6 - 48\pi a_s G m^3 \mathrm{M}^6 \hbar^2}}{2 G m^3 \mathrm{M}^3} \end{Bmatrix}$$

The condition that the potential barrier $\boldsymbol{r_h}$ **disappears:** $r_h = r_b$



$$r_{hb} = \frac{2\sqrt{3\pi}\sqrt{a_s}\hbar}{\sqrt{G}m^{3/2}}$$ …fully determined from microscopic quantities

The mass inside r_{hb} forms BH ↓

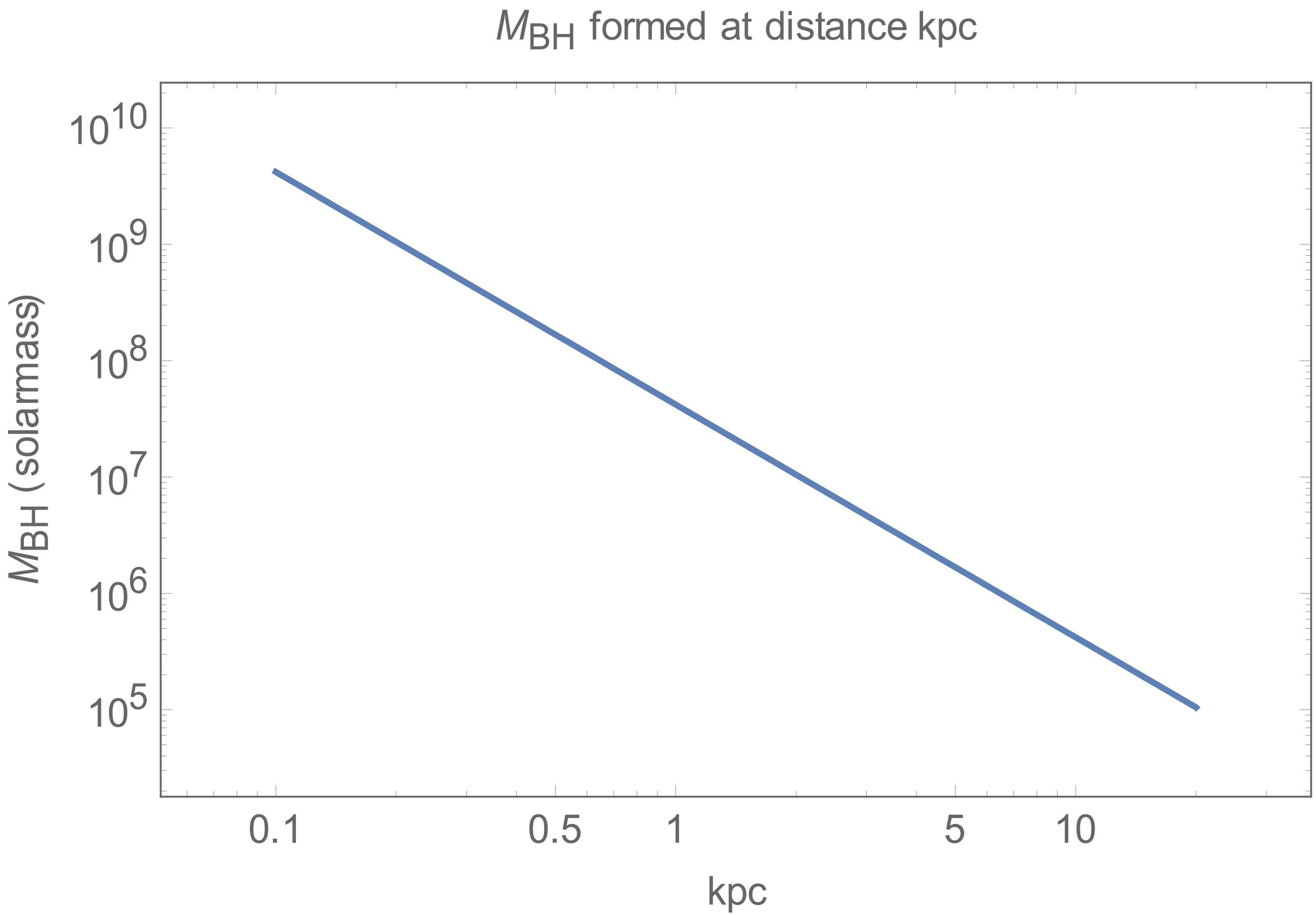
M_{BH}/M_{DH}

$$= \frac{8\sqrt{3}\pi^{3/2}a_s{}^{3/2}\hbar^3}{G^{3/2}m^{9/2}R r_0{}^2}$$

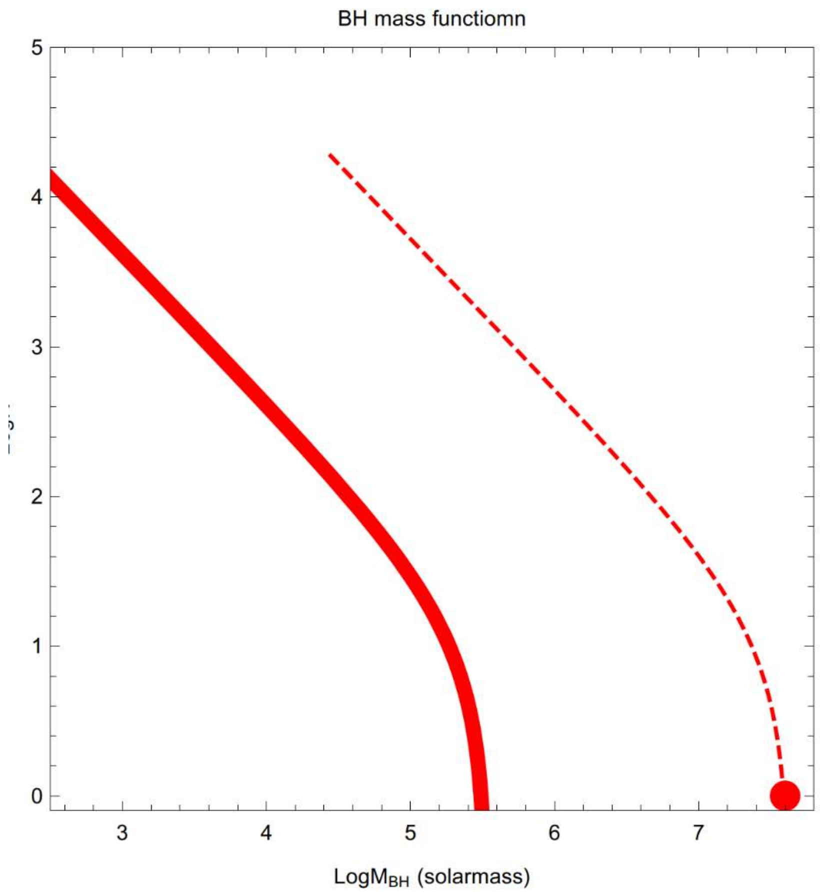
Typically $m = \frac{\text{eV}}{10^5 c^2}, a_s = \frac{\text{Meter}}{10^{29}}$, r0 = kpc, R = 10kpc,, $M = 10^{12} M_{sun}$

$\Rightarrow$ r_{hb} = 108 pc, $\frac{M_{BH}}{M_{DH}} = 4.2 * 10^{-5}$, $t_{BHform} = 6.25 * 10^4$year

- However, many BHs defined by the radius r_{hb} everywhere in the galaxy $\Rightarrow$ **BH everywhere**



- However actually, provided $a_s = 10^{-30.4}\text{meter}$, centrally concentrated



10^3 BHs coalescent to form a big SMBH within the time scale

$$t_d = \frac{v^3}{G^2 n\, m^2\, lnL} \approx \frac{N}{lnN} t_{ff} \approx 6.8 \times 10^7 year$$

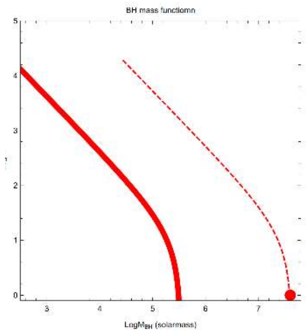
and many BHs of mass $10^5 - 10\text{M}_{sun}$ maybe formed at outer region.

7. Conclusions and Discussions

- Based on the scenario that SMBH is formed before stars, we considered SMBH formation form BEC(DE/DM) collapse
- GP equation, Gaussian approximation →V_{eff}
- Angular momentum controls SMBH-DH ratio.
- Attractive force by Axion balances with the angular momentum

$$\rightarrow \frac{M_{SMBH}}{M_{DH}} \approx 10^{-5}$$

- BH formation in all scales



DE→DM→SMBH…dark species are connected with each other

- DE → SMBH → stars and galaxies…The whole dynamics